Exercise 9.1:

1) Find the value of the following determinants, where i = $\sqrt{-1}$

i) $\begin{vmatrix} 2i & -3i \\ i^3 & -2i^5 \end{vmatrix}$

ii) $\begin{vmatrix} 1+3i & i-2 \\ -i-2^3 & 1-3i \end{vmatrix}$

2) Show that:

i) $\begin{vmatrix} \cos\theta & \cos\phi \\ \sin\theta & \sin\phi \end{vmatrix} = -\sin(\theta-\phi)$

ii) $\begin{vmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{vmatrix} = 1$

3) Find x, if

i) $\begin{vmatrix} 2 & x \\ -4 & 3 \end{vmatrix} = 0$

ii) $\begin{vmatrix} x & -3 \\ -1 & x+2 \end{vmatrix} = 0$

4) Prove that

i) $2\begin{vmatrix} 8 & -5 \\ -2 & 6 \end{vmatrix} = \begin{vmatrix} 14 & -4 \\ -2 & 6 \end{vmatrix}$

ii) $4\begin{vmatrix} x & y \\ 3x & 0 \end{vmatrix} = 3\begin{vmatrix} x & 0 \\ 5x & -4y \end{vmatrix}$

5) Evaluate the following determinants.

i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

ii) $\begin{vmatrix} 1 & 2 & 3 \\ 12 & 13 & 14 \\ 33 & 34 & 35 \end{vmatrix}$

iii) $\begin{vmatrix} 1 & 1 & 1 \\ 10 & 11 & 12 \\ 100 & 101 & 102 \end{vmatrix}$

iv) $\begin{vmatrix} 0 & 1 & -5 \\ 2 & 1 & -3 \\ -3 & 1 & -4 \end{vmatrix}$

v) $\begin{vmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{vmatrix}$

6) Prove that

$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$

7) Show that

$\begin{vmatrix} i & -2i & -1 \\ 3i & i^3 & -2 \\ 1 & -3 & -i \end{vmatrix} = 11i$, where i= $\sqrt{-1}$

8) Prove that $\begin{vmatrix} a & a & a \\ a & b & b \\ a & b & c \end{vmatrix} = a(b-c)(a-b)$

Hence find the value of $\begin{vmatrix} 3 & 3 & 3 \\ 3 & 5 & 5 \\ 3 & 5 & 7 \end{vmatrix}$.

9) Find x, if

i) $\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$

ii) $\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix} = 29$

iii) $\begin{vmatrix} x & 2 & 1 \\ 3 & x & -2 \\ 1 & 3 & 1 \end{vmatrix} = 5$

iv) $\begin{vmatrix} 0 & -3 & x \\ x+1 & 3 & 1 \\ 4 & 1 & 5 \end{vmatrix} = 0$

v) $\begin{vmatrix} x & 1 & 2 \\ 3 & x & 3 \\ 1 & 2 & 2 \end{vmatrix} = 3$

Exercise 9.2:

(1) Find the value of

i) $\begin{vmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ -1 & -1 & 0 \end{vmatrix}$

ii) $\begin{vmatrix} 10 & 57 & 107 \\ 12 & 64 & 124 \\ 15 & 78 & 153 \end{vmatrix}$

iii) $\begin{vmatrix} 1 & 18 & 72 \\ 2 & 40 & 148 \\ 2 & 45 & 150 \end{vmatrix}$

iv) $\begin{vmatrix} 10 & 10 & 10 \\ 213 & 211 & 210 \\ 372 & 375 & 377 \end{vmatrix}$

v) $\begin{vmatrix} 1 & 1001 & 17 \\ 3 & 3003 & 19 \\ 5 & 5005 & 23 \end{vmatrix}$

(2) Without expanding, show that the value of the following determinants is zero.

i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$

ii) $\begin{vmatrix} x+a & x-y & x+c \\ y+a & y+b & y+c \\ z+a & z+b & z+c \end{vmatrix}$

iii) $\begin{vmatrix} 1 & 1 & x \\ 1 & x & x^2 \\ 1 & x^2 & x^3 \end{vmatrix}$

iv) $\begin{vmatrix} 0 & x-y & y-z \\ y-x & 0 & z-x \\ z-y & x-z & 0 \end{vmatrix}$

v) $\begin{vmatrix} 1 & xy & xy(x+y) \\ 1 & yz & yz(y+z) \\ 1 & zx & zx(z+x) \end{vmatrix}$

vi) $\begin{vmatrix} 2 & 3 & 3^3 \\ 2^2 & 3^2 & 3^4 \\ 2^3 & 3^3 & 3^5 \end{vmatrix}$

Solve the following problems using Graphical Method.

1) Maximize $Z = 25x_1 + 20x_2$
   Subject to constraints
   $3x_1 + 2x_2 \leq 1800$
   $2x_1 + 7x_2 \leq 1400$
   $0 \leq x_1 \leq 350$
   $0 \leq x_2 \leq 150$
   $X_1, x_2 \geq 0$

2) Maximize $Z = 0.08x_1 + 0.10x_2$
   Subject to constraints
   $x_1 + x_2 \geq 15000$
   $x_1 \geq 3000$
   $x_2 \geq 5000$
   $X_1, x_2 \geq 0$

3) Maximize $Z = x_1 + x_2$
   Subject to constraints
   $3x_1 + 2x_2 \leq 1$
   $5x_1 + 8x_2 \leq 600$
   $x_1 \geq 0, x_2 \geq 0$

4) Maximize $Z = 6x_1 + 10x_2$
   Subject to constraints
   $3x_1 + 5x_2 \geq 10$
   $2x_1 + 3x_2 \leq 18$
   $x_1, x_2 \geq 0$

5) Max $Z = 40x_1 + 35x_2$
   Subject to
   $2x_1 + 3x_2 \leq 60$
   $4x_1 + 3x_2 \geq 96$
   $X_1, x_2 \geq 0$

6) Max $Z = 5x_1 + 7x_2$
   Subject to $x_1 + x_2 \leq 4$
   $3X_1 + 8x_2 \leq 24$
   $10x_1 + 7x_2 \leq 35$
   $X_1, x_2 \geq 0$

7) Max $Z = 6x_1 + 4x_2$
   subject $2x_1 + 3x_2 \leq 30$
   $3x_1 + 2x_2 \leq 24$
   $X_1 + x_2 \geq 3$
   $X_1, x_2 \geq 0$

EXERCISE 2.1

1)

i) State the order and the type of the following matrices.

a) $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 5 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ -3 \\ 4 \\ 9 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

e) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

ii) Write the elements a_{11}, a_{31}, a_{33}, a_{24} and a_{23} (whichever exists) for the following matrices.

a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 3 & 2 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

e) $\begin{bmatrix} p & q & r & s \\ a & b & c & d \\ e & f & g & h \\ m & n & l & k \end{bmatrix}$ f) $\begin{bmatrix} a & b \\ -a & -b \\ c & d \\ e & g \end{bmatrix}$

2) Find a matrix A iff $A = [a_{ij}]_{3x2}$ such the $a_{ij} = \frac{1}{2}(i-2j)$ for all i and j

3) Write eh transpose of the following matrices.

i) $\begin{bmatrix} 2 & 0 & 5 & 9 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & 2 \\ 0 & 9 \\ -1 & 5 \end{bmatrix}$

iv) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 9 & 5 \end{bmatrix}$

v) $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 9 & 3 \\ -1 & 2 & 5 \\ 2 & 3 & 4 \end{bmatrix}$

4) State the types of the matrix A = $\begin{bmatrix} 3 & 9 & 5 \\ 9 & 4 & 0 \\ 5 & 0 & -2 \end{bmatrix}$. Find A. What is your observation? What can you conclude?

5) Give examples of two matrices of order 2 x 2 which are symmetric.

6) Give examples of two matrices of order 3 x 3 which are skew-symmetric.

7) Determine whether the following matrices are singular or non-singular.

i) $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

iii) $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

v) $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 2 \end{bmatrix}$ vi) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

EXERCISE 2.2

1) For the following matrices find the given scalar multiplications.

i) A = $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 0 & 6 \end{bmatrix}$ find 2A.

ii) B = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ find 3B

iii) C = $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ find -4C.

2) If A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ find

i) A + B

ii) B – A

iii) A + B + C

iv) (C – B) - A

3) If A = $\begin{bmatrix} 2 & 2 & 2 \\ 2 & -1 & -3 \\ 1 & 0 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 3 & 3 \\ 3 & 0 & 5 \\ 6 & 9 & -1 \end{bmatrix}$ C = $\begin{bmatrix} 4 & 4 & 4 \\ 5 & -1 & 0 \\ 7 & 8 & -1 \end{bmatrix}$ then find

i) 2A – B + C ii) A + C – 3B

iii) 2C – B + 3A

4) Find X and Y if

i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$, $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

ii) $2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$, $3X-2Y=\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

5) Find the following of x and y if

i) $\chi\begin{bmatrix} 2 \\ 3 \end{bmatrix}+\gamma\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 1 0 \\ 5 \end{bmatrix}$

ii) $2\begin{bmatrix} 1 & 3 \\ 0 & \chi \end{bmatrix}+\begin{bmatrix} \gamma & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

6) Find the values of x, y, z and w if

$$2\begin{bmatrix} \chi & \gamma \\ z & w \end{bmatrix}=\begin{bmatrix} \chi & 6 \\ -1 & w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

7) If A = $\begin{bmatrix} 2 & 0 & 2 \\ -3 & 4 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$, find a matrix C such that A + B + C is a zero matrix.

8) Find the following products.

i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

iv) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

v) $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

vi) $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

vii) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

9) If A = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, C = $\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ find AB and AC. What is your observation?

10) If A = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}_{3x2}$, B = $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}_{2x2}$, C = $\begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}_{2x2}$ verifying that (AB) C = A (BC)

11) Simplify the following. $\left\{3\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & 3\end{bmatrix}-\begin{bmatrix}1 & 5 & -2\\ -3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$

12) If $A=\begin{bmatrix}1 & 3 & 3\\ 3 & 1 & 3\\ 3 & 3 & 1\end{bmatrix}$ then show that $A^2 - 5A$ is a scalar matrix.

13) If $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & -3\\ 1 & -1 & 0\end{bmatrix}$ find $A^2 - 5A + 61$.

14) If $A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}$ find a matrix X such that $AX=I_2$.

15) If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$

16) If $A=\begin{bmatrix}-3 & 2\\ 2 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1 & x\\ y & 0\end{bmatrix}$ and $(A+B)(A-B) = A^2 - B^2$ then find x and y.

EXERCISE 2.3

1) Find which of the following matrices are invertible.

i) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ ii) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

iii) $\begin{bmatrix}1 & 2\\ 3 & 3\end{bmatrix}$ iv) $\begin{bmatrix}2 & 3\\ 10 & 15\end{bmatrix}$

vii) $\begin{bmatrix}3 & 4 & 3\\ 1 & 1 & 0\\ 1 & 4 & 5\end{bmatrix}$ viii) $\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 3\\ 1 & 2 & 3\end{bmatrix}$

ix) $\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{bmatrix}$

2) Find AB if $A=\begin{bmatrix}1 & 2 & 3\\ 1 & -2 & -3\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 1 & 2\\ 1 & -2\end{bmatrix}$. Hence determine if AB has inverse.

3) If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and X is a 2 x 2 matrix such that AX=I then find X.

4) If $A=\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$ then show that $A^2 - 4A + I = 0$. Hence find A^{-1}.

EXERCISE 2.4

1. Find the co-factors of the elements of the following matrices.

1) $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

2. Find the matrix of co-factors for the following matrices.

1) $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

3. Find the adjoint of the following matrices.

1) A=$\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

4. If A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that

A(adjA)=(adj A) A= A I.

5. Find the inverse of each of the following matrices by the adjoint method.

i) $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

iii) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ iv) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

EXERCISE 2.5

1) Solve the following equations by the adjoint method
   i) X+2y=2, 2x+3y=3
   ii) X+y=4, 2x-y=5
   iii) 2x+6y=8, x+3y=5
   iv) 2x-y=-2, 3x+4y=3
   v) 5x-y+4z=5, 2x+3y+5z=2 and 5x-2y+6z=-1
2) Solve the following equations by the adjoint method.
   i) 2x+y=5, 3x+5y=-3
   ii) X+3y=2, 3x+5y=4
   iii) 3x-y=1, 4x+y=6
   iv) 5x+2y=4, 7x+3y=5
   v) x-y+z=1, 2x-y=1 and 3x+3y-4z=2

vi) $x+y=1$, $y+z=5/3$, $z+x=4/3$

vii) $x+y+z=6$, $3x-y+3z=10$ and $5x+y-4z=3$

viii) $2x-y+z=1$, $x+2y+3z=8$ and $3x+y-4z=1$

1) In a class there are 40 boys and 20 girls. A teacher wants to select class monitors. In how many was can this be done if either a boy or a girl is selected as a class monitor?
2) There are 4 candidates for an English scholarship, 3 candidates for a History scholarship and 2 candidates for a Sanskrit scholarship. In how many ways any one of these scholarships be awarded?
3) Given 4 flags of different colors, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?
4) Find the number of 4-lettered words, with or without meaning, that can be formed out of the letters of the word TIME, where the repetitions of the letters i) is not allowed ii) is allowed.
5) How many 3 digit numbers can be formed from the digits 0, 2, 4, 5, 7 if the repetition of the digits i) is not allowed ii) is allowed?
6) How many 3 digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?
7) A letter lock contains 3 ring containing 5 different letters. Determine the maximum number of false trials that can be made before the lock outstanding opened?
8) For set of 5 true/false questions, no student has written all correct answers
   and no two students have given the same sequence of answers. What is the maximum number of students in the class for this to be possible/
9) How many numbers are there between 100 and 1000, such that 6 is in the unit's place?
10) How many numbers between 100 and 1000 are such that exactly one of the digits is 6?
11) How many natural numbers not exceeding 4321 can be formed with the digits 1, 2, 3, 4 if the repetition of digits in a number is not allowed?
12) How many even numbers greater than 300 can be formed with the digits 1,2,3,4,5 if repetition of digits in a number is not allowed?
13) How many numbers each lying between 9 and 1000 can be formed with the digits 0,1,2,3,7,8 if repetition of the digit in a number is allowed?
14) There are 2 gates to enter a school and 3 staircases to go from first floor to the second floor. How many possible ways are there for a student to enter the school and go to the classroom on the second floor and come back?
15) How many five digit numbers can be formed using the digits 0,1,2,3,4 and 5, which are divisible by 3 without repeting the digits.

1) How many 4 – letter words with or without meaning, can be formed using the letters of the word RUCHIKA if
   (i) Repetition of letters is not allowed.
   (ii) Repetition of letters is allowed.
2) Determine how many words can be formed using the letters of the word LOGARITHM, If
   (i) Vowels are always together.
   (ii) No tow vowels are together.
   (iii) Consonants occupy even positions.
   (iv) Begin with O and end with T.
3) In how many ways can the letters of the word STORY be arranged, so that
   (i) T and Y are always together.
   (ii) T is always next to Y.
4) How many arrangements of the letters of the word 'COMRADE' can be made, if the relative positions of vowels and consonants are not changed.
5) How many different 4-digit numbers can be formed using the digits 2, 4, 5, 6, 7, 8, if

(i) Repetition of digits is not allowed.
(ii) Repetition of digits is allowed.

6) How many number between 100 and 100 can be formed with the digit 0,1,2,3,4,5, if the repetition of the digits is not allowed?
7) How many numbers with 6-digits can be formed from the digit 3,4,5,6,7,8; if no digit is repeated. How many of these are
   (i) Divisible by 5?
   (ii) Not divisible by 5?
8) A code word should consist of two English Capital alphabets followed by two distinct digits from 1 to 9 e.g. MH 23 is a code word.
   (i) How many such code words are available?
   (ii) How many of them end with an even integer?

1) Find the number of distinct permutations of the letters of the following words
   (i) DIVIJA
   (ii) SARASWATI
   (iii) REPRESENTATION
   (iv) COMBINATION.
2) There are three identical books on English, 4 identical books on Hindi, 2 identical books on MATHEMATICS. In how many distinct ways can they be arranged on a shelf?
3) There are 5 Red, 4 White and 3 Green marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles are drawn, determine the number of different arrangements.
4) How many different arrangements can be made, with the letters of the word MATHEMATICS? In how many of these arrangements, vowels occur together?
5) How many different words can be formed with the letters of the word DIPAWALI? In how many of these
   (i) P and D are never together?
   (ii) Vowels are always together?
6) In how many different ways can the letters on the word MUMBAI be arranged so that A is always next to B?
7) In how many different ways can the letters of the word CONSTITUTION be arranged? How many of these will have the letter N, both at the beginning and at the end?
8) How many different words can be formed with the letters Fisher's the word INDIA? In how many of these
   (i) Two I's are always together?
   (ii) N and A are always together?
9) In how many different ways can the letters of the word ' SALOON' be arranged
   (i) If the tow O's must not come together.
   (ii) If the consonants and vowels must occupy alternate places.

1) Find the value of
   (i) $^{14}C_5$
   (ii) $^{90}C_2$
   (iii) $^{20}C_4+^{20}C_5$
   (iv) $^{31}C_{26} - ^{30}C_{26}$
2) Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 7 blue balls if each selection consists of 3 balls of each colour.
3) In how many ways can a team of 3 boys and 2 girls be selected from 6 boys and 5 girls?
4) At the end of a certain meeting, every one hand shaken hands with every one else, it was found that 45 handshakes were exchanged how many members were present at the meeting?

5) How many chords can be drawn through 21 points on a circle?
6) Find the maximum number of diagonals that can be drawn in an n-sided polygon. Also find number of diagonals if
   (i) n = 12 sides
   (ii) n = 15 sides
   (iii) decagon

1) Find n, if ${}^{n}C_7 = {}^{n}C_{11}$
2) Find n, if ${}^{18}C_{3n} = {}^{18}C2_{n+3}$
3) Find n, if ${}^{18}C_{2r} = {}^{18}Cn^2+3$
4) Find n, if ${}^{2n}C_r = {}^{2n}C_{r+2}$
5) Find n, if ${}^{n}C_{n-4} = 15$
6) Find x, if ${}^{n}C_r = x\,{}^{n}P_r$
7) Find r, if ${}^{15}C_4 + {}^{15}C_5 + {}^{16}C_6 + {}^{17}C_7 = {}^{18}C_r$
8) Find the value of ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$
9) Find the difference between the greatest values of
   (i) ${}^{13}C_r - {}^{11}C_r$
   (ii) ${}^{14}C_r - {}^{8}C_r$
   (iii) ${}^{15}C_r - {}^{12}C_r$
10) In how many ways a man can invite 6 friends to a party so that two or more of remain present?
11) A group consists of 8 men and 5 women. Find the number of committees of 5 persons that can be formed, if committee consists of at least 3 women.
12) A committee of 12 persons is to be formed from 9 women and 8 men. In how many ways can this be done if at least five women have to be included in a committee? In how many of these committees
   (i) Women are in majority.
   (ii) Men are in majority.
13) A question paper consists of 11 questions divided into two sections I and II. Section I consists of 5 questions and section II consists of 6 questions taking at least 2 questions from each section?
14) Among 22 cricket players, there are 3 wicket keepers and 6 bowlers. In how many ways can a team of 11 players be chosen so as to include exactly one wicket keeper and at least 4 bowlers?
15) In how many ways can 5 students be selected out of 11 students if
   (i) 2 particular students are included?
   (ii) 2 particular students are not included?

Warehouse		Market				
		I	**II**	**III**	**IV**	**Supply**
	A	5	2	4	3	**22**
	B	4	8	1	6	**15**
	C	4	6	7	5	**8**
Requirement		**7**	**12**	**17**	**9**	

		Sales depots			
Availability		S_1	S_2	S_3	**Demand**
Factories	F_1	6	6	1	**10**
	F_2	-2	-2	-4	**150**
	F_3	3	2	2	**50**
	F_4	8	8	3	**100**
Demand		**80**	**120**	**150**	

	Warehouses			
Plants	**A**	**B**	**C**	**Production**
X	8	16	16	**152**
Y	32	48	32	**164**
Z	16	32	48	**154**
Demand	**144**	**204**	**82**	

	Outlets				
Plants	A	B	C	D	**Capacity of Production**
X	4	6	8	6	**700**
Y	3	5	2	5	**400**
Z	3	9	6	5	**600**
Requirement	**400**	**450**	**350**	**500**	**1,700**

	Customers				
Factory	A	B	C	D	**Supply**
P	40	25	22	33	**100**
Q	44	35	30	30	**30**
R	38	38	28	30	**70**
Demand	**40**	**20**	**60**	**30**	

	Plants	**A**	**B**	**C**	**Total Demand**
Sales outlets					
X	3	9	6	6	**20**
Y	4	4	6	6	**40**
Z	8	3	5	5	**60**
Total supply	**40**	**50**	**30**	**30**	120

Note: Solve each example by using all three methods