MEASURES OF CENTRAL TENDENCY

INTRODUCTION:

Often, one encounters the term "Measures of central tendency" in a book on statistics (or an examination). One may also find mention of the term in a description of summary statistics.

What is a measure of central tendency?

Simply put, it is a value that tries to summarize a given dataset. It does so by providing a central (or middle) value that (sort of) represents the entire dataset.

Imagine the marks scored by 10 students in an examination were as follows:
20 20 20 20 20 20 20 20 20 20

If I asked you about the performance of the students, you could say that "student 1 scored 20 marks; student 2 scored 20 marks; ……… student 10 scored 20 marks". The problem with this strategy is encountered when we have to deal with large datasets, of say, 100 values or more. It would take considerable time and effort to describe the entire dataset.

Therefore, people tried to find a way by which they could convey the essence of the dataset. The measures of central tendency do just that.

It is like describing a person in one word- "smart"/ "creative", etc. In fact, we summarize things on a routine basis, often without realizing it, whether it is movies, books, classes, or just about anything. Coming back to the example, one could simply say "the students scored 20 marks on average".

This average value represents the entire dataset, and gives us an idea about the performance of the students.

The most common measures of central tendency are the Mean, Median and Mode.

MEANING AND DEFINITION OF AVERAGE:
(MEASURES OF CENTRAL TENDENCY)

A single value that describes the characteristics of the entire series is called the 'Central Value' or an 'Average'. It is the most representative value of the series.

DEFINATIONS:

According to **CROXTON and COWDE**" "*an Average value is a single value within the range of the data that is used to represent all of the value in the series. Since average is somewhere within the range of data, it is also called the measures of central tendency.*"

According to **E.A. Waugh**, "*an average is a single value from a group of values to represent them in some way – a value which is supposed to stand for whole group of which it is a part, a typical of all the values in the group.*"

According to **Prof. Clark**, "*Average is an attempt to find one single figure to describe whole figures.*"

Thus an average represents the whole series and it's value lies in between maximum and minimum values. It is generally located in the centre or middle of the distribution.

ARITHMETIC MEAN (A.M.)

It is the most popular and widely used measures of central tendency.

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DEFINITION: An Arithmetic mean is defined as the value obtained by dividing the sum of all the observations by the number of items. It is also known a the Arithmetic Mean. It is denoted by $\bar{x}$ Arithmetic mean can be calculated by the formula.

$$\text{Mean} = \frac{\text{Sum of all values of observations}}{\text{Total number of observations}}$$

$$\text{Symbolically, } \bar{x} = \frac{\sum x}{n}$$

Where, $\bar{x}$ = Arithmetic mean
$\sum x$ = Sum of all observations
n = Number of items

CALCULATION OF ARITHMETIC MEAN:

(A) Individual Data (Distribution):

To calculate mean, add all the values of given observation and divide it by the number of items i.e.

$$\text{A.M.} = \frac{\text{Sum of all values of items}}{\text{Total number of items}}$$

$$\text{Symbolically, } \bar{x} = \frac{\sum x}{n} \text{ Or } = \frac{x_1 + x_2 + x_3 + - - - - + x_n}{n}$$

Where, $\bar{x}$ = Arithmetic mean
$\sum x$ = Sum of all items
n = number of items

Example:

(1) The following are the daily wages of 5 workers in a factory. Calculate mean wage.

Daily Wage (Rs.):	500	600	700	800	900

Solution: $\bar{x} = \frac{\sum x}{n} = \frac{500+600+700+800+900}{5} = \frac{3500}{5} = 700$

$\therefore$ **Mean Wage = Rs. 700/-**

(2) Marks of 8 students in an Economics test is given below. Find the mean mark of the students.

Marks: 50, 60, 80, 90, 75, 45, 55, 65

Solution: $\bar{x} = \frac{\sum x}{n} = \frac{50+60+80+90+75+45+55+65}{8} = \frac{520}{8} = 65$

$\therefore$ **Mean Marks = 65**

(3) The following are the monthly income of 10 families in a village. Calculate mean average income per family.

Income (Rs.): 550, 650, 700, 400, 550, 650, 800, 900, 450, 350.

Solution: $\bar{x} = \frac{\sum x}{n} = \frac{550+650+700+400+550+650+800+900+450+350}{6}$

$= \frac{6000}{6} = 600$

$\therefore$ **Average Income = (Rs.) 600/-**

(4) Calculate Arithmetic Mean of the following marks obtained by 10 students.

Marks: 80,	75,	65,	45,	85,	60,	50,	70,	40,	90

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Solution: Arithmetic Mean $\bar{x} = \frac{\sum x}{n} = \frac{80+75+65+45+85+60+50+70+40+90}{10}$

$= \frac{660}{10} = 66$

$\therefore$**Average Marks = 66**

(5) Calculate Arithmetic Mean form the following data:
Data: 3, 6, 5, 4, 7, 9, 9, 5, 7, 9, 8, 6, 5, 3, 8, 6

Solution: $\bar{x} = \frac{\sum x}{n} = \frac{3+6+5+4+7+9+9+5+7+9+8+6+5+3+8+6}{16} = \frac{100}{16} = 6.25$

$\therefore \bar{x}$ **= 6.25**

(B) CALCULATION OF ARITHMETIC MEAN: (DISCRETE SERIES DISCRETE DISTRIBUTION or UN GROUPED DATA):

In Discrete series, Arithmetic Mean can be calculated in any of the two ways:
(1) Direct Methods
(2) Short Cut Method

(1) DIRECT METHOD: It is used when values are small and few.

Formula $\bar{x} = \frac{\sum fx}{n}$

$\bar{x}$ = Arithmetic mean,
F = frequency of an item,
X = Value of item,
$n = \sum f$

Steps:

(1) Multiply the frequency of each item with respective variable and add them to get Efx

(2) Divide $\sum fx$ by the total frequency Income & Expenditure, $(\sum f)$ to get $\bar{x}$.

Q.1. Marks obtained by 50 students in a class is given below. Calculate Arithmetic Mean by Direct Method

Marks	10	20	30	40	50
No. of Students	5	15	10	15	5

Solution:

Marks (x)	**No. of Students**	fx
10	05	50
20	15	300
30	10	300
40	15	600
50	05	250
	$N=\sum f=50$	$\sum fx=1500$

$\therefore \bar{x}$ **=** $\frac{\sum fx}{N} = \frac{1500}{500}$ **= 30**

$\therefore$ **Average Marks = 30**

Q. 2. Calculate Arithmetic mean from the following

Marks	5	6	7	8	9
No. of Students	5	12	15	10	8

Solution:

Marks (x)	**No. of Students**	fx

5	05	25
6	12	72
7	15	105
8	10	80
9	08	72
	N=$\sum f$=50	$\sum f$x=354

$\therefore \bar{x} = \frac{\sum fx}{N} = \frac{354}{50} = 7.8$

$\therefore$ **Mean Marks = 7.8**

Q. 3. Daily wages of 25 workers in a sugar factory is given below. Calculate mean wages of workers.

Solution:

Wages (Rs.)	50	60	70	80	90
No. of Workers	4	6	3	7	5

Solution:

Wages (x) (Rs.)	**No. of Workers (f)**	**fx**
50	4	200
60	6	360
70	3	210
80	7	560
90	5	450
	N=$\sum f$=25	$\sum f$x=1780

$\therefore \bar{x} = \frac{\sum fx}{N} = \frac{1780}{25} = 71.20$

$\therefore$ **Mean Marks = 71.20**

(2) Short-cut Method: It is used to avoid mistakes in calculation when the values are large and high.

Formula: $\bar{x} = A + \left(\frac{\sum fd}{N}\right)$

Where, A = Assumed mean (Any figure form 1st figure from 1st column can be taken as assumed mean) N= total frequency – i.e. $\sum f$, d = deviation – i.e.(x-A)

Steps:

(1) Select Assumed mean

(2) Find out deviation of item from assumed mean – i.e. d=x-A

(3) Multiply these deviations with the respective frequency and take total – i.e. $\sum f$d.

(4) Apply the formula: $\bar{x} = A + \left(\frac{\sum fd}{N}\right)$

Q.1. Height of 50 students in a class is given below. Find mena height of student (use Short cut method)

Height (cm)	110	120	130	140	150
No. of Students	5	12	15	10	8

Solution:

Height (x)	**No. of Students(f)**	**D=X-A**	**fd**
110	05	-20	-100
120	12	-10	120

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130 A	15	0	0
140	10	10	100
150	08	20	160
	N=$\sum f$=50	$\sum f$x=40	

$\therefore \bar{x} = A + \left(\frac{\sum fd}{N}\right) = 130 + \left(\frac{40}{50}\right) = 130 + 0.8 = \mathbf{130.80}$

∴ Mean Height of a Student = 130.80

Q. 2. Marks obtained by 100 students in a class I given below. Calculate Arithmetic Mean using short cut method.

Height (cm)	110	120	130	140	150
No. of Students	5	12	15	10	8

Solution:

Height (x)	**No. of Students(f)**	**D=X-A**	**fd**
10	12	-20	-240
20	28	-10	-280
30 A	35	0	0
40	13	10	130
50	12	20	240
	N = $\sum f$ = 100		$\sum f$d = -150

$\therefore \bar{x} = A + \left(\frac{\sum fd}{N}\right) = 130 + \left(\frac{-150}{100}\right) = 130 + (-1.5) = \mathbf{28.50}$

∴ Mean Mark = 28.50

(C) CALCULATION OF ARITHMETIC MEAN: CONTINUOUS SERIES: (GROUPED DATA)

In continuous series, Exact value of variables are not known. Only the range is given. So, the midpoints of the various classes are taken as the representative of that particular class. Arithmetic mean can be calculated using any of the following method:

(i) Direct Method:

Formula: $\bar{x} = \frac{\sum fx}{N}$

Where, $\bar{x}$ = Arithmetic mean, X = Midpoint of the classes,
f = frequency of each class, N = Total frequency – i.e. $\sum f$

$$\text{Midpoint} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Steps:

(1) Obtain midpoint of each class and denote it by 'X' –
(2) Multiply midpoint by respective frequency of class and added them up to get $\sum f$x
(3) Divide $\sum f$x by the sum of frequencies i.e. – N or $\sum f$.

Q. 1. Calculate Arithmetic mean from the following Data: (use Direct Method)

No. of Product Sold	0-10	10-20	20-30	30-40	40-50	50-60
No. of Salesman	3	17	21	19	23	7

Solution:

No. of Product Sold (Mid Value 'X')	**No. of Salesmen (f)**	**(fx)**

05	03	15
15	17	255
25	21	525
35	19	665
45	23	1035
55	07	385
	$N=\sum f=90$	$\sum fx = 2880$

$$\therefore \bar{x} = \frac{\sum fx}{N} = \frac{2880}{90} = \mathbf{32}$$

Q. 2. Calculate Arithmetic Mean from the following data: (use Direct Method)

Marks:	0-10	10-20	20-30	30-40	40-50
No. of Students:	5	30	35	20	10

Solution:

Marks (Mid X)	**No. of Students (f)**	**(fx)**
05	05	25
15	30	450
25	35	875
35	20	700
45	10	450
$N = \sum f = 100$		$\sum fx = 2500$

$$\therefore \bar{x} = \frac{\sum fx}{N} = \frac{2500}{100} = \mathbf{25}$$

(ii) Short Cut Method:

Formula: $\bar{x} = A + \left(\frac{\sum fd}{N}\right)$

Where, $\bar{x}$ = Arithmetic Mean;
A = Assumed Mean
d = Deviation of mid point from Assumed mean A (i.e. X – A)
N = Total frequency (i.e. $\sum f$)
f = Frequency of the class.

Steps:

(1) Take assumed mean (A) from the mid points.
(2) Obtain deviation of midpoint from the Assumed Mean – i.e. d = X-A
(3) Multiply deviation by respective frequencies and sum up it to get $\sum f$d.
(4) Find total frequency N = $\sum f$
(5) Apply the formula: $\bar{x} = A + \left(\frac{\sum fd}{N}\right)$

Example

Q.1. Calculate Arithmetic mean from the following data (use Short Cut Method).

Wages Per Day (Rs.)	10-20	20-30	30-40	40-50	50-60
No. of Workers	25	34	40	29	22

Solution:

Wages per day Mid 'x'	**No. of Workers (f)**	**Deviation D= X-A**	**fd**	
15	25	-20	-500	

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25	34	-10	-340	840
35 A	40	0	0	
45	29	10	290	
55	22	20	440	730
	$N=\sum f=150$		$\sum fd$ = -110	

$\therefore \bar{x} = A + \left(\frac{\sum fd}{N}\right)$ **=** $35 + \frac{-110}{150}$ = 35+(-0.73) = **34.27**

Q. 2. Marks obtained by 100 students is given below. Calculate Arithmetic mean for the data (Use short cut method.)

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	30	35	20	10

Solution:

Marks Mid 'x'	**No. of Students**	**Deviation D= X-A**	f**d**
05	05	-20	-100
15	30	-10	-300
25 A	35	0	0
35	20	10	200
45	10	20	200
	$N=\sum f=100$		$\sum fd$ = 0

$\therefore \bar{x} = A + \left(\frac{\sum fd}{N}\right)$ **=** $25 + \frac{0}{100}$ = **25**

(iii) Step – Deviation Method: It is the shortest method for calculation of Arithmetic Mean. in this method, each deviation (d=x-A) is divided by the common factor (i)

Formula: $\bar{x} = A + \left(\frac{\sum fd_1}{N} x\, i\right)$

Where, $\bar{x}$ = Arithmetic Mean. A = Assumed Mean
N = Total frequency. d = X-A
d_1 = X-A/i. X = Mid Point
i = Common factor (width of class interval)
$\sum f d_1$ = Product of deviation multiplied by their respective frequencies.

Steps:

(1) Obtain Mid Point.
(2) Take Assumed Mean.
(3) Calculate deviation from Assumed mean.
(4) Divide deviation by common factor.
(5) Multiply deviation by respective frequencies.
(6) Sum up the product of frequencies and step deviation.
(7) Apply the formula: $\bar{x} = A + \left(\frac{\sum fd_1}{N} x\, i\right)$

Q.1. Calculate Arithmetic Mean from the following data by Step Deviation Method.

Class	10-20	20-30	30-40	40-50	50-60
Frequency	2	3	9	4	2

Solution:

Class (Mid X)	Frequency (f)	Deviation D= X-A (A=35)	$d_1 = \frac{X-A}{i}$ (i=10)	Product fd_1
15	2	-20	-2	-4
25	3	-10	-1	-3
A **35**	9	0	0	0
45	4	10	1	4
55	2	20	2	4
	$N=\sum f=20$		$\sum fd_1=01$	

$$\therefore \quad \bar{x} = A + \left(\frac{\sum fd_1}{N} x i\right) \qquad = 35 + \left(\frac{1}{20} x10\right) = 35+0.5 = \mathbf{35.5}$$

Q. 2. Calculate Mean from the following Data (Use Step Deviation Method)

Class	10-20	20-30	30-40	40-50	50-60
Frequency	2	3	9	4	2

Solution:

Class (Mid X)	Frequency (f)	Deviation D= X-A (A=35)	$d_1 = \frac{X-A}{i}$ (i=10)	Product fd_1
25	12	-30	-3	-36
35	10	-20	-2	-20
45	13	-10	-1	-13
55 A	25	0	0	0
65	20	10	1	20
75	30	20	2	60
	$N=\sum f=110$			$\sum fd=11$

$$\therefore \quad \bar{x} = A + \left(\frac{\sum fd_1}{N} x i\right) \qquad = 55 + \left(\frac{11}{110} x110\right) = 55+1 = \mathbf{56}$$

(D) MERITS OF ARITHMETIC MEAN:

- **Simple:** Definition (meaning) of Arithmetic Mean is simple, precise or exact.
- **Easy:** It is easy to calculate and understand.
- **Rigidly DEFINED:** Arithmetic Mean is rigidly defined so, the result obtained will remain same, whatever method may be used.
- **Arrangement of Data – not Necessary**: It is not necessary to arrange data in ascending or descending order, to calculate Arithmetic Mean.
- **Based on all Items:** Arithmetic Mean is based each and every item of the series.
- **Stability:** It is reliable and sufficiently stable and is not much affected by the fluctuation in samples.
- **All Details not Required:** Only the data is required to calculate the mean. Other details are not required.
- **Algebraic Treatment:** Arithmetic Mean is capable of further Algebraic manipulation.
- **Centre of Gravity:** It balances the values on either side of it. So, it is the centre of Gravity.

(E) DEMERITS OF ARITHMETIC MEAN:

- **Distribution of items:** Arithmetic means do not equate the entire distribution of items.
- **Affected by Extreme items:** Value of Arithmetic mean is affected by extreme items (i.e., biggest or smallest). It reduces it's utility as a representative value. **For example:** If the age of

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Grandfather is 85 years and the age of 3 grandsons are 10, 8 and 7 respectively, then average age of them is 27.5 years.

- **Not Workable:** If data is incomplete or if any single item is missed or ignored in the series, mean cannot be calculate. It may loose it's accuracy.
- **Absurd Results:** Sometimes, it gives absurd result. For example: if there are 2 children in a family and 3 in another, then, average number of child in these families is 2.5. It is absurd. It cannot be in fraction.
- Cannot be found by inspection. It is difficult to locate mean b inspection.
- **Value may not be in series:** Exact value of mean may not be in series. **Example:** Mean of 10, 20, 30 and 40 is 25.
- **Open-end classes:** A.M. is based on assumptions. It cannot be calculated effectively for open-end classes.
- **Not fruitful:** It cannot be used for qualitative and deceptive study.

COMBINE MEAN:

If $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic mean of two samples of sizes n_1 and n_2 respectively then, the arithmetic mean $\bar{x}$ of the distribution combining the two can be calculated as

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

This formula can be extended for still more groups or samples.

$$\bar{x}_1 = \frac{\sum x_{1i}}{n_1} \Rightarrow \sum x_{1i} = n_1 \bar{x}_1$$

Justification: $\bar{x}_1 = \frac{\sum x_{1i}}{n_1} \Rightarrow \sum x_{1i} = n_1 \bar{x}_1$ = total of the observations of the first sample

Similarly $\sum x_{2i} = n_2 \bar{x}_2$ = total of the observations of the first sample

The combined mean of the two samples

$$= \frac{\text{combined total}}{n_1 + n_2}$$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

Example The average marks of three batches of students having 70, 50 and 30 students respectively are 50, 55 and 45. Find the average marks of all the 150 students, taken together.

Solution :

Let x be the average marks of all 150 students taken together.

	Batch - I	Batch - II	Batch - III
	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$
A. marks :	= 50	= 55	= 45
No. of students	n_1 = 70	n_2 = 50	n_3 = 30

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3}{n_1 + n_2 + n_3} = \frac{70 \times 50 + 50 \times 55 + 30 \times 45}{70 + 50 + 30} = \frac{7600}{150}$$

$$\bar{x} = 50.67 \text{ marks}$$

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Example The mean of a certain number of observations is 40. If two or more items with values 50 and 64 are added to this data, the mean rises to 42. Find the number of items in the original data.

Solution:

Let 'n' be the number of observations whose mean $\bar{x}$ = 40.

$\bar{x} = \frac{\sum x_i}{n} \therefore \sum x_i = n\bar{x} = n(40) = 40n$ total of n values.

Two more items of values 50 and 64 are added therefore, total of (n + 2) values :

$= \sum x_i + 50 + 64$

$= 40n + 50 + 64$

$= 40n + 114$

Now new mean is 42.

New $\bar{x} = \frac{\text{New total of } (n+2) \text{ values}}{n+2}$

$42 = \frac{40n + 14}{n+2}$

42n + 84 = 40n + 114

2n = 30

n = 15

Therefore, the number of items in the original data = 15.

Example The sum of deviations of a certain numbers of observations measured from 4 is 72 and the sum of deviations of observations measured from 7 is -3. Find the number of observations and their mean.

Solution:

Let 'n' be the required number of observations $\sum(x_i - 4) = 72$, therefore, $\sum x_i - 4n = 72$

......Note $\sum 4 = 4n$ and $\sum(x_i - 7) = -3$ therefore,

$\sum x_i - 7n = -3$ Subtracting the two equations we get,

$\sum x_i - 4n = 72$

$\sum x_i - 7n = -3$

(-) (+) (+)

3n=75 n = 25

Putting n = 25 in $\sum x_i - 4n = 72$, we get

$\sum x_i - 4(25) = 72$

$\sum x_i = 72 + 100$

$\sum x_i = 172$

Now Mean is given by $\bar{x} = \frac{\sum x_i}{n} = \frac{172}{25} = 6.88$

What is the 'Geometric Mean':

The geometric mean is the average of a set of products, the calculation of which is commonly used to determine the performance results of an investment or portfolio. Technically defined as "the 'n'th root product of 'n' numbers", the formula for calculating geometric mean is most easily written as:

Where 'n' represents the number of returns in the series.
The geometric mean must be used when working with percentages (which are derived from values), whereas the standard arithmetic mean will work with the values themselves.

DEFINITION of 'Harmonic Average'

The mean of a set of positive variables. Calculated by dividing the number of observations by the reciprocal of each number in the series.

MODE:

Mode is that value which occurs most frequently in a set of observations. It is that item which repeats maximum number of items. It is the value having maximum, (highest) frequency. It is also known as 'Modal Value' or 'Norm of the series.'

(A) CALCULATION OF MODE: INDIVIDUAL SERIES (DATA):
Steps: (1) Arrange the data in ascending or descending order. **(2)** Use tally mark and make frequency table. **(3)** Mode is the value – repeated maximum number of times.

Example:
(1) Find mode from the following data:
5, 10, 12, 10, 11, 12, 10, 13, 11, 10, 21, 30
Solution: Arrange Data in ascending order and make frequency table

Observation	Tally Mark	Frequency
05	I	01
10	IIII	04
11	II	02
12	II	02
13	I	01
21	I	01
30	I	01
		Total 12

In this observation 10 has repeated maximums (i.e. 4) times.
∴ **Mode = 10**

Q. 2. Find mode from the following data:
65, 40, 48, 52,54, 56, 52, 48, 49, 52, 40, 42, 52, 60, 61, 52, 70, 71, 52, 48

Observation	Tally Mark	Frequency
40	I	02
42	II	01
48	III	03
49	I	01

52	IIIII	06
54	I	01
56	I	01
60	I	01
61	I	01
65	I	01
70	I	01
71	I	01
		Total = 20

In this, observation 52 has repeated maximum (i.e. 6) times ∴ **Mode = 52**.

(B) CALCULATION OF MODE; DISCRETE SERIES: (UNGROUPED DATA):

In case of Discrete series, mode can be located by Inspection. It is the value in the series, having maximum frequency.

Example:

(1) Calculate mode from the following Data:

Size of Shirt	10	20	30	40	50	60	70
No. of Persons	4	3	2	6	8	5	1

Solution:

Size of Shirt	10	20	30	40	50	60	70
No. of Persons	4	3	2	6	8	5	1

In this, the observation 50 has maximum frequency – (i.e. 8). Mode is the value having maximum frequency.

∴ **Mode = 50**

(2) Calculate mode form the following frequency distribution:

Daily Wages (in Rs.)	67	68	69	73	79	87	93	95	96
No. of Workers	20	100	120	160	90	95	80	75	30

Solution:

Daily Wages (in Rs.)	67	68	69	73	79	87	93	95	96
No. of Workers	20	100	120	160	90	95	80	75	30

In this, the observation 73 has the highest frequency – (i.e. 160). Mode is the value having maximum frequency.

∴ **Mode = 73**

(C) CALCULATION OF MODES:
CONTITUOUS SERIES (GROUPED DATA)

Steps:

(1) Find the class interval having maximum frequency (model class)

(2) Use interpolation formula.

Mode = $L_1 + \frac{(L_2 + L_1)(f_1 - f_0)}{2f_1 + f_0 - f_2}$ find exact value of mode

Where, L_1 = Lower limit of the class
L_2 = Upper limit of the class
f_1 = Frequency of modal class (highest frequency)
f_0 = Frequency of the class preceding modal class.
f_2 = Frequency of the class succeeding modal class

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Examples:

Q. 1. Calculate mode from the following data:

Sales (in Rs. crores)	0-4	4-8	8-12	12-16	16-20	20-24
No. of Firms	4	6	12	7	6	3

Solution:

Sales (in Rs. Crores)	**No. of Firms**
0-4	4
4-8	6-f_0
8-12	12-f_1
12-14	7-f_2
16-20	6
20-24	8

In this, 12 is the highest frequency.
Corresponding modal class = 8-12

$$\therefore \text{Mode} = L_1 + \frac{(L_2 + L_1)(f_1 - f_0)}{2f_1 + f_0 - f_2}$$

$$= 8 + \frac{(12-8)(12-6)}{(2x12) - 6 - 7}$$

$$= 8 + \left(\frac{4x6}{24-13}\right) = 8 + \left(\frac{24}{11}\right)$$

$= 8+2.18$ **= 10.18**

$\therefore$ **Mode = 10.18**

Q. 2. Find mode from the following data:

Daily Wages	10-20	20-30	30-40	40-50	50-60
No. of Workers	15	20	25	10	5

Solution:

Daily Wages (Rs.)	**No. of Workers**
10-20	15
20-30	20-f_0
30-40	25-f_1
40-50	10-f_2
50-60	5

In this, 25 is the highest frequency.
Corresponding modal class = 30-40

$$\therefore \text{Mode} = L_1 + \frac{(L_2 + L_1)(f_1 - f_0)}{2f_1 + f_0 - f_2}$$

$$= 30 + \frac{(40-30)x(25-20)}{2(25) - 20 - 10}$$

$$= 30 + \left(\frac{10x5}{50-30}\right) = 30 + \left(\frac{50}{20}\right)$$

$= 30+2.5$

$\therefore$ **Mode = 32.5**

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(D) MERITS AND DEMERITS OF MODE:

(a) Merits of Mode:

- **Easy and Simple:** Mode is simple to understand and easy to calculate.
- **Not Influenced by Extreme Items:** Extreme items has no effect on mode.
- **All Detail no required:** To calculate mode, all details of items are not required. Only the norm or most frequent item is to be known.
- **Value Exist in Series:** Mostly, Modal Value exists in series.
- **Most Representative Average:** Mode is the most descriptive average in use. **Example:** Model height, Modal wage, Average number of Railway accidents in an year. It is also useful for qualitative data.
- **Graphical Location:** Mode can be determined graphically – through Histograms.
- **Useful for Open – end Classes:** It's value can be determined in open-end classes.
- **Inspection:** Mode can be determined by inspection. It is the most frequent observation in the series.

(E) DEMERITS OF MODE:

- **Ambiguous Definition:** Mode is Illustration-defined and indefinite. It is not properly defined. There can be more than one Mode in a series. **Example:** (Bimodal tri-modal). If the values do not repeat, then Mode is absent.
- **Not Based on all Items:** Mode is not based on each and every item of the series. It consider only most frequent item.
- **Not Algebraic Properties:** Mode is not capable of further Algebraic Manipulations.
- **Not a Good Measure:** Mode is not a good measure of central tendency as it consider only the modal class and ignore other variables in the series.
- **Limited Scope:** In small sample scope of Mode is limited. When non of the variable repeats, Mode is absent.
- **Unstable Average:** Mode is the most unstable average. So it is difficult to determine it's true value.

MEDIAN:

Median is the middle – most value in the series – arranged in ascending or descending order. It divides the series into 2 equal parts. It is a positional average.

(A) CALCULATION OF MEDIAN: INDIVIDUAL DATA:

(a) If the number of items in the distribution is odd (uneven)

Steps:

(1) Arrange data in ascending or descending order.

(2) Apply the formula: $M = \text{Size of } \frac{(n+1)^{th}}{2}$ item

Where, M = Median n = Numbers of items

Q. 1. Marks obtained by 7 students in a class test is given below. Find Mark.

Marks	7, 10, 11, 5, 8, 12, 15

Solution:

Step: Arrange Data in ascending order.

Marks	5, 7, 8, 10, 11, 12, 15

$\text{Median} = \text{Size of } \frac{(n+1)^{th}}{2}$ item

$= \text{Size of } \frac{(7+1)^{th}}{2}$ item

$= \text{Size of } 4^{th}$ item

Corresponding 4^{th} Value in the series is 10.

$\therefore$ **Median = 10**

Q. 2. Find Median from the following data:

35, 38, 40, 39, 35, 36, 37

Solution: Arrange Data in ascending order.

35, 35, 36, 37, 38, 39, 40

Median = Size of $\frac{(n+1)^{th}}{2}$ item

= Size of $\frac{(7+1)^{th}}{2}$ item

= Size of 4^{th} item

Corresponding 4^{th} Value in the series is 37.

∴ **Median = 37**

(b) If the number of items in the distribution is even:

(1) Arrange Data in Ascending or Descending order.

(2) Apply the formula: M = Value of $\frac{(n+1)^{th}}{2}$ item

It will be in traction. So, add values before and after the fractional value and divided total by 2 – to get Median value.

Q. 1. Suppose 8 Students secured following marks in Economics: Find Median Marks.

83, 47, 59, 25, 36, 30, 92, 74

Solution: Arrange Data in ascending order.

25, 30, 36, 47, 59, 74, 83, 92

Median = Size of $\frac{(n+1)^{th}}{2}$ item

= Size of $\frac{(8+1)^{th}}{2}$ item

= Size of 4.5^{th} item

$= \frac{\text{Size of } 4^{th} \text{ item} + \text{size of } 5^{th} \text{ item}}{2}$

$= \frac{47+59}{2} = \frac{106}{2} = \mathbf{53}$

∴ **Median Marks = 53**

Q. 2. Find Median from the following data:

14, 13, 12, 11, 15, 16, 18, 17, 19, 20

Solution: Arrange Data in ascending order.

11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Median = Size of $\frac{(n+1)^{th}}{2}$ item

= Size of $\frac{(10+1)^{th}}{2}$ item

= Size of 5.5^{th} item

$= \frac{\text{Size of } 5^{th} \text{ item} + \text{size of } 6^{th} \text{ item}}{2}$

$= \frac{15+16}{2} = \frac{31}{2} = \mathbf{15.5}$

∴ **Median = 15.5**

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(B) CALCULATION OF MEDIAN: DISCRETE SERIES (UNGROUPED DATA):

Steps:

(1) Arrange Data in Ascending order (if not arranged) and write corresponding frequency.

(2) Find cumulative frequency (f) – by successively adding the frequencies of each item in the series.

(3) Apply the formula: Value of $\frac{(N+1)^{th}}{2}$ item. Where, N = $\sum f$.

(4) Look C.f. and find the value equal to or greater than $\frac{(N+1)^{th}}{2}$ item.

(5) Value corresponding to that C.f. will be Median.

Q. 1 Find Median Marks obtained by the students:

Marks:	5	10	15	20	25	30
No. of Students:	10	4	5	12	9	8

Solution:

Marks	**No. of Students (f)**	**C.f.**
05	10	10
10	04	14
15	05	19
20	12	**31**
25	09	40
30	08	48
	N=$\sum f$=48	

$\therefore$Median = Size of $\frac{(N+1)^{th}}{2}$ item

= Size of $\frac{(48+1)^{th}}{2}$ item

= Size of 24.5^{th} item (It lies in Cf 31)

The value corresponding to the Cf 31 is : 20

$\therefore$ **Median = 20**

Hence, Median Marks = 20

Q. 2 Find Median from the following data:

Marks:	13	12	11	14	15
No. of Students:	13	8	12	6	10

Solution:

Marks	**No. of Students (f)**	**C.f.**
11	12	12
12	08	20
13	13	**33**
14	06	39
15	10	49
	N=$\sum f$=49	

$\therefore$Median = Size of $\frac{(N+1)^{th}}{2}$ item

= Size of $\frac{(49+1)^{th}}{2}$ item

= Size of 25^{th} item (It lies in Cf 33)

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The value corresponding to the Cf 33 is : 13
∴ **Median = 13**

Q. 3 Find Median from the following data:

Income (Rs.)	1000	2000	3000	4000	5000	6000
No. of Persons	20	10	15	25	12	8

Solution:

Income (Rs.)	**No. of Persons (f)**	**C.f.**
1,000	20	20
2,000	10	30
3,000	15	45
4,000	25	70
5,000	12	82
6,000	08	90
	N=$\sum f$=90	

$$\therefore \text{Median} = \text{Size of } \frac{(N+1)^{th}}{2} \text{ item}$$

$$= \text{Size of } \frac{(90+1)^{th}}{2} \text{ item}$$

= Size of 45.5^{th} item (It lies in Cf 70)
The value corresponding to the Cf 70 is : 4,000
∴ **Median Income = Rs. 4,000**

Q. 4 Find Median from the following data:

Size (X)	5	6	7	8	9	10	11	12	13
Frequency (f)	48	52	56	60	63	57	55	50	52

Solution:

Size (X)	**Frequency (f)**	**C.f.**
05	48	48
06	52	100
07	56	156
08	60	216
09	63	279
10	57	336
11	55	391
12	50	441
13	52	493
		N=$\sum f$=493

$$\therefore \text{Median} = \text{Size of } \frac{(N+1)^{th}}{2} \text{ item}$$

$$= \text{Size of } \frac{(493+1)^{th}}{2} \text{ item}$$

= Size of 247^{th} item (It lies in Cf 279)
The value corresponding to the Cf 279 is : 9
∴ **Median = 9**

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(C) CALULATION OF MEDIAN: CONTINUOUS SERIES (GROUPED DATA)

Steps:

(1) Arrange data in ascending or descending order (if not) and write frequencies of respective class.

(2) Calculate cumulative frequencies (C.f.)

(3) Determine the class in which Median Class lies: (using the formula)

Rank of Median = Size of $\left(\frac{N}{2}\right)^{th}$ item, where N = $\sum f$

(4) To get Exact value of the median, use the interpolation, formula:

$$M = L_1 + \frac{\left(\frac{N}{2} - C.f.\right)}{f} X\left(L_2 - L_1\right) \textbf{ OR } M = L_1 + \frac{\left(\frac{N}{2} - C.f.\right)}{f} X f$$

Where, M = Median

L_1 = Lower limit of the Median Class

L_2 = Upper limit of the Median Class

f = Frequency of the Median Class

C.f. = Cumulative frequency of the class preceding the Median Class

i = (L_2 – L_1)

Example:

Q. 1. Calculate median from the following data:

Income (Rs.)	0-10	10-20	20-30	30-40	40-50	50-60
No. of Persons	3	6	8	10	8	5

Solution:

Income (Rs.)	**No. of Persons (f)**	**C.f.**
0-10	03	03
10-20	06	09
20-30	08	17
30-40	10	**27**
40-50	08	35
50-60	05	40
		N=$\sum f$=40

Rank of Median = Size of $\left(\frac{N}{2}\right)^{th}$ item

= Size of $\left(\frac{40}{2}\right)^{th}$ item

= Size of 20^{th} item (It lies in Cf 27)

Corresponding median Class = 30-40

L_1 = 30, L_2 = 40, f_1 = 10, Cf. = 17, i=10, $\frac{N}{2} = \frac{40}{2}$ = 20

$$\therefore\ M = L_1 + \frac{\left(\frac{N}{2} - C.f.\right)}{f} X i \quad \textbf{=} = 30 + \left(\frac{20-17}{10} X10\right)$$

$$=30+\left(\frac{03}{10}X10\right)=30+03=\mathbf{33}$$

∴ Median Income = Rs. 33/-

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Q. 2. Calculate median from the following data:

Working Hours (per week)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
No. of Workers	5	6	15	10	5	4	3	2

Solution:

Income (Rs.)	No. of Persons (f)	C.f.
5-10	05	5
10-15	06	11
15-20	15	**26**
20-25	10	36
25-30	05	41
30-35	04	45
35-40	03	48
40-45	02	50
	$N=\sum f=50$	

Rank of Median = Size of $\left(\frac{N}{2}\right)^{th}$ item = Size of $\left(\frac{50}{2}\right)^{th}$ item

= Size of 25^{th} item (It lies in Cf 26)

Corresponding median Class = **15-20**

$L_1 = 15$, $L_2 = 20$, $f = 15$, Cf. = 11, i=5, $\frac{N}{2} = \frac{50}{2} = 25$

$\therefore M = L_1 + \frac{\left(\frac{N}{2} - C.f.\right)}{f} X i$ **=** $= 15 + \left(\frac{25-11}{15} X5\right)$

$=15+\left(\frac{14}{15}X5\right)$ $=15+4.67=$**19.67**

Q. 3. Calculate median from the following data:

Wages per hours (Rs.)	0-10	10-20	20-30	30-40	40-50
No. of Workers	5	6	15	10	5

Solution:

Wages per hours (Rs.)	No. of Workers (f)	C.f.
0-10	05	05
10-20	10	15
20-30	12	**27**
30-40	15	42
40-50	08	50
	$N=\sum f=50$	

Rank of Median = Size of $\left(\frac{N}{2}\right)^{th}$ item = Size of $\left(\frac{50}{2}\right)^{th}$ item

= Size of 25^{th} item (It lies in Cf 27)

Corresponding median Class = **20-30**

$L_1 = 20$, $L_2 = 30$, $f = 12$, Cf. = 15, i=10, $\frac{N}{2} = \frac{50}{2} = 25$

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$\therefore\ M = L_1 + \dfrac{\left(\frac{N}{2} - C.f.\right)}{f} X i$ **=** $= 20 + \left(\dfrac{25-15}{12} X5\right)$

$=20+\left(\dfrac{10}{12}X10\right)$ $=20+8.33=$**28.33**

Q. 4. Calculate median from the following data:

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of Students	4	6	10	16	12	8	4

Solution:

Marks	**No. of Students (f)**	**C.f.**
0-5	04	4
5-10	06	10
10-15	10	20
15-20	16	**36**
20-25	12	48
25-30	08	56
30-35	04	60
	$N=\sum f=60$	

Rank of Median = Size of $\left(\dfrac{N}{2}\right)^{th}$ item = Size of $\left(\dfrac{60}{2}\right)^{th}$ item

= Size of 30^{th} item (It lies in Cf 36)

Corresponding median Class = **15-20**

$\therefore\ M = L_1 + \dfrac{\left(\frac{N}{2} - C.f.\right)}{f} X f$ **=** $= 15 + \left(\dfrac{30-20}{16} X5\right)$

$=15+\left(\dfrac{10}{16}X5\right)$ $=15+3.125=$**18.125 or 18.13**

Q. 5. Calculate median from the following data:

Income (Rs.)	0-10	10-20	20-30	30-40	40-50	50-60
No. of Persons	3	5	7	13	7	5

Solution:

Income (Rs.)	**No. of persons (f)**	**C.f.**
0-10	03	03
10-20	05	08
20-30	07	15
30-40	13	**28**
40-50	07	35
50-60	05	40
	$N=\sum f=40$	

Rank of Median = Size of $\left(\dfrac{N}{2}\right)^{th}$ item = Size of $\left(\dfrac{40}{2}\right)^{th}$ item

= Size of 20^{th} item (It lies in Cf -28)

Corresponding median Class = **30-40**

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$\therefore\ M = L_1 + \dfrac{\left(\dfrac{N}{2} - C.f.\right)}{f} X i$ $\quad = \; = 30 + \left(\dfrac{20-15}{13} X10\right)$

$=30+\left(\dfrac{5}{13}X10\right)$ $\quad =30+3.846$ $\quad =$**33.85**

(D) MERITS OF MEDIAN:

- **Easy:** Median is simple to understand and easy to calculate.
- **Inspection:** In some cases, median can be located by mere inspection.
- **Rigidly defined:** Median has well defined formula.
- **All Details – Not Required:** Details about the values are not required to calculate median. It can be determined even if complete data is not available.
- **Not Influenced by Extreme Items:** It is not affected by the value of extreme (end) items in the series.
- **Useful to Study Attributes:** Median is an ideal average to study qualitative characteristics (attributes) like intelligence, honesty, health, etc.
- **Exist in Series:** Mostly, the median value exist within the series.
- **Graphical Location:** Median can also be determined graphically.

(E) DEMERITS OF MEDIAN:

- **Array is Essential:** It is necessary to arrange data in ascending or descending order, to calculate Median. If the series is lengthy, the task of finding the median is difficult.
- **Not Precise:** It cannot be precisely determined in a series with even number of items. In such a case, the value estimated may not be the actual value in the series.
- **Not Based on All Observations:** Median is a positional average. So, its value is not based on all the observations in the series.
- **Ignores Extreme ITEMS:** Since the median ignore extreme items, it is not useful in those cases where large weights are to be given to extreme items.
- **Not capable of Algebraic Manipulation:** It is not capable of further algebraic treatment.
- **Not a True Representative:** Median is not actual (true) representative value in the series, as it ignore the extreme items. It is not a representative especially when the distribution lacks uniformity.
- **Sampling Fluctuations:** Value of median is more affected by sampling fluctuations.

QUARTILES

Definition: Quartiles are the values (data) which divide the series (distribution) into four equal parts. They are the 3 values that divide the distribution into 4 part, each representing one quarters of the score. These 3 values are called as first quartile (Q_1), second quartile (Q_2) and third quartile (Q_3). Second quartile is nothing but the median.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
Q_1 $\quad Q_2$ $\quad Q_3$

The figure shows that quartiles are the three points: Q_1, Q_2, and Q_3 which divided the series into four parts in such a way that each part contains equal number of items.

(A) FIRST AND THIRD QUARTILES FOR INDIVIDUAL DATA:

Steps:

(1) Let 'n' observations of Individual Data be arranged in ascending order.
(2) Apply the formula's First Quartile (Q_1)

$= \text{value of} \left(\dfrac{n+1}{4}\right)^{th}$ Observation.

Second Quartile (Q_2)

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= value of $2\left(\frac{n+1}{4}\right)^{th}$ Observation.

Third Quartile (Q_3)

= value of $3\left(\frac{n+1}{4}\right)^{th}$ Observation.

Q. 1. Find the first and third quartiles from the following data.

4, 6, 7, 9, 11, 5, 8, 10, 14, 12, 13

Solution: After arranging the given data in ascending order we get,

4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

n = number of observations ∴ n = 11

First quartile (Q_1)

= value of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $\left(\frac{11+1}{4}\right)^{th}$ observation.

= value of 3^{rd} observation.

Ans.: Q_1 = 6

Third quartile (Q_3)

= value of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $3\left(\frac{11+1}{4}\right)^{th}$ observation.

= value of (3 x 3)th observation

= value of 9^{th} observation.

Ans.: Q_3 = 12

Q. 2. Find the first and third quartiles from the following data.

30, 35, 37, 32. 36, 33, 38, 31, 34, 39, 40

Solution: After arranging the given data in ascending order we get,

30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40

n = number of observations ∴ n = 11

First quartile (Q_1)

= value of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $\left(\frac{11+1}{4}\right)^{th}$ observation.

= value of 3^{rd} observation.

Ans.: Q_1 = 32

Third quartile (Q_3)

= value of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $3\left(\frac{11+1}{4}\right)^{th}$ observation.

= value of $(3 \text{ x } 3)^{th}$ observation

= value of 9^{th} observation.

Ans.: Q_3 = 38

Q. 3. Calculate the first quartile, second quartile and third quartiles from the following data.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Solution: After arranging the given data in ascending order we get,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

n = number of observations ∴ n = 15

First quartile (Q_1)

$= \text{value of } \left(\frac{n+1}{4}\right)^{th} \text{ observation.}$

$= \text{value of } \left(\frac{15+1}{4}\right)^{th} \text{ observation.}$

= value of 4^{th} observation.

Ans.: Q_1 = 4

Second quartile (Q_2)

$= \text{value of } 2\left(\frac{n+1}{4}\right)^{th} \text{ observation.}$

$= \text{value of } 2\left(\frac{15+1}{4}\right)^{th} \text{ observation.}$

= value of 8^{th} observation.

Ans.: Q_2 = 8

Third quartile (Q_3)

$= \text{value of } 3\left(\frac{n+1}{4}\right)^{th} \text{ observation.}$

$= \text{value of } 3\left(\frac{15+1}{4}\right)^{th} \text{ observation.}$

= value of 12^{th} observation.

Ans.: Q_3 = 12

Q. 4. Find out the first and third quartile from the following daily pocket money of eleven students.

Rs. 50, 40, 60, 90, 80, 70, 100, 140, 130, 120, 110

Solution: After arranging the given data in ascending order we get,

40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140

n = number of observations ∴ n = 11

First quartile (Q_1)

$= \text{value of } \left(\frac{n+1}{4}\right)^{th} \text{ observation.}$

$= \text{value of } \left(\frac{11+1}{4}\right)^{th} \text{ observation.}$

= value of 3^{rd} observation.

Ans.: Q_1 = 60

Third quartile (Q_3)

$= \text{value of } 3\left(\frac{n+1}{4}\right)^{th} \text{ observation.}$

= value of $3\left(\frac{11+1}{4}\right)^{th}$ observation.

= value of $(3 \times 3)^{th}$ observation

= value of 9^{th} observation.

Ans.: Q_3 = 120

Q. 5 Find the first and third quartiles from the following data.

13, 16, 25, 30, 32, 45, 65

Solution: After arranging the given data in ascending order we get,

13, 16, 25, 30, 32, 45, 65

First quartile (Q_1)

= value of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $\left(\frac{7+1}{4}\right)^{th}$ observation.

= value of 2^{nd} observation.

Ans.: Q_1 = 16

Third quartile (Q_3)

= value of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= value of $3\left(\frac{7+1}{4}\right)^{th}$ observation.

= value of $(3 \times 2)^{th}$ observation

= value of 6^{th} observation.

Ans.: Q_3 = 45

(B) FIRST AND THIRD QUARTILES FOR DISCRETE DISTRIBUTION (UNGROUPED DATA):

Steps:

(1) Arrange data in ascending order.

(2) Calculate the cumulative frequency's.

(3) Apply the following formula for first and third quartiles.

Q_1 = value of $\left(\frac{N+1}{4}\right)^{th}$ Observation

Q_3 = value of $3\left(\frac{N+1}{4}\right)^{th}$ Observation

Where, $N = \sum f$

For Example:

Q.1: Find the first and third quartile for the following ungrouped data.

X	5	4	9	12	15	6	10
F	6	8	12	8	6	9	10

Solution:

X	f	**C.f**
04	06	06
05	08	14

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06	09	23
09	12	35
10	10	45
12	08	53
15	06	59
	N=59	

Q_1 = the size of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= the size of $\left(\frac{59+1}{4}\right)^{th}$ observation.

= the size of 15^{th} observation

Ans.: Q_1 = 6

Q_3 = the size of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= the size of $3\left(\frac{7+1}{4}\right)^{th}$ observation.

= the size of 6^{th} observation

Ans.: Q_3 = 12

Q.2: Find the first and third quartile for the following ungrouped data.

X	2	3	4	5	6	7	8	9	10
F	2	4	6	8	10	12	10	14	13

Solution:

X	f	**C.**f
02	02	02
03	04	06
04	06	12
06	10	30
07	12	42
08	10	52
09	14	66
10	13	79
	N=79	

Q_1 = the size of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= the size of $\left(\frac{79+1}{4}\right)^{th}$ observation.

= the size of 20^{th} observation

Ans.: Q_1 = 5

Q_3 = the size of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= the size of $3\left(\frac{79+1}{4}\right)^{th}$ observation.

= the size of 60^{th} observation

Ans.: $Q_3 = 9$

(C) FIRST AND THIRD QUARTILES FOR CONTINUOUS FREQUENCY DISTRIBUTION (UNGROUPED DATA):

First and third quartiles for continuous frequency distribution are calculated by following formula.

$$Q_1 = L_1 + \left(\frac{\frac{N}{4} - C.f}{f}\right) \times i$$

Where, L_1 = Lower limit of Q_1 class.
$i = L_2 - L_1$.
N = Total number of frequencies (i.e. $\sum f$)
C.f = Cumulative frequency of class preceding in Q_1 class.
f = frequency of Q_1 class.

$$Q_3 = L_1 + \left(\frac{\frac{3N}{4} - C.f}{f}\right) \times i$$

Where, L_1 = Lower limit of Q_3 class.
$i = L_2 - L_1$.
N = Total number of frequencies (i.e. $\sum f$)
C.f = Cumulative frequency of class preceding in Q_3 class.
f = frequency of Q_3 class.

For Example:

Q. 1: Find first and third quartile from the following data (distribution)

C.I.	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	10	15	25	40	35	20	5

Solution:

C.I.	**Frequency**	**C.f.**
10-20	10	10
20-30	15	25
30-40	25	50
40-50	40	90
50-60	35	125
60-70	20	145
70-80	05	150
	N = 150	

N = Total number of observation = 150

$\frac{N}{4} = \frac{150}{4} = 37.5$

$\frac{3N}{4} = 3 \times 37.5 = 112.5$

Hence,
30-40 is Q_1 class
50-60 is Q_3 class

For Q_1

$L_1 = 30$, C.f = 25, f=25, i=10

$$Q_1 = L_1 + \left(\frac{\frac{N}{4} - C.f}{f}\right) \times i$$

$$= 30 + \left(\frac{\frac{37.5}{4} - 25}{25}\right) \times i$$

$$= 30 + \frac{12.5}{25} \times 10 = 30 + \frac{125}{25} = 30+5$$

Ans: $Q_1 = 35$

For Q_3

$L_1 = 50$, $C.f = 90$, $f=35$, $i=10$

$$Q_3 = L_1 + \left(\frac{\frac{3N}{4} - C.f}{7}\right) \times i$$

$$= 50 + \left(\frac{11.5 - 90}{35}\right) \times 10$$

$$= 50 + \frac{22.5}{35} \times 10 = 50 + \frac{225}{35} = 50+6.43$$

Ans: $Q_3 = 56.43$

Q. 2. Calculate Q_1 and Q_3 for the following data.

C.I.	0-10	10-20	20-30	30-40	40-50
f	7	10	15	8	5

Solution:

C.I.	f	**C.f.**
0-10	07	07
10-20	10	17
20-30	15	32
30-40	08	40
40-50	05	45
	N = 45	

N = Total number of observation = 45

$$\frac{N}{4} = \frac{45}{4} = 11.25$$

$$\frac{3N}{4} = 3 \times 11.25 = 33.75$$

Hence,

10-20 is Q_1 class

30-40 is Q_3 class

For Q_1

$L_1 = 10$, $C.f = 07$, $f=10$, $i=10$

$$Q_1 = L_1 + \left(\frac{\frac{N}{4} - C.f}{f}\right) \times i$$

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$= 30+\left(\frac{11.25-7}{10}\right) \times 10$

$= 10 + 11.25 - 7$

$= 10 + 4.25$

Ans: $Q_1 = 14.25$

For Q_3 :

$L_1 = 30$, $C.f = 32$, $f= 8$, $i=10$

$$Q_3 = L_1 + \left(\frac{\frac{3N}{4} - C.f}{7}\right) \times i$$

$= 30 + \left(\frac{33.75-32}{8}\right) \times 10$

$= 30+\frac{1.75}{8} \times 10$

$= 30 + \frac{17.5}{8} = 30+2.18$

Ans: $Q_3 = 32.18$

DECILES:

Definition: Deciles divide the series into ten equal parts. Thus, there must be nine points which will divide the arrayed series in such a way that each part contains an equal numbers of items. The value of these none points are called deciles.

They are denoted as D_1, D_2, D_3,.......... D_9. There are nine deciles in a distribution. The 5^{th} decile D_5 being the median. While calculating these values in individual and discrete values in the series, we add all observations i.e. 1^{st} to n^{th} (total numbers). For calculating D_1 to D_9 we use the following formula:

(1) First Decile: D_1 = size of $\left(\frac{N+1}{10}\right)^{th}$ item,

(2) Second Decile: D_2 = size of $2\left(\frac{N+1}{10}\right)^{th}$ item,

(3) Third Decile: D_3 = size of $3\left(\frac{N+1}{10}\right)^{th}$ item,

(4) Fourth Decile: D_4 = size of $4\left(\frac{N+1}{10}\right)^{th}$ item

(5) Fifth Decile: D_5 = size of $5\left(\frac{N+1}{10}\right)^{th}$ item

(6) Sixth Decile: D_6 = size of $6\left(\frac{N+1}{10}\right)^{th}$ item

(7) Seventh Decile: D_7 = size of $7\left(\frac{N+1}{10}\right)^{th}$ item

(8) Eight Decile: D_8 = size of $8\left(\frac{N+1}{10}\right)^{th}$ item

(9) Ninth Decile: D_9 = size of $9\left(\frac{N+1}{10}\right)^{th}$ item

(A) INDIVIDUAL DATA:

Steps:

(1) Arrange data in ascending order.

(2) Apply the formula for given decile.

Q. 1. Find D_2, D_4 and D_7 for the following data.

11, 15, 12, 16, 13, 17, 14, 18, 20, 19, 25, 21, 24, 22, 23, 29, 28, 27, 26

Solution: Arranging the values in ascending order, we get,

11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29

The number of Value N = 19

Second Decile:

Hence, Rank of D_2,

$= 2\left(\frac{N+1}{10}\right)^{th}$ item

$= 2\left(\frac{19+1}{10}\right)^{th}$ item

$= (2 \text{ X } 2)^{th}$ item

$= 4^{th}$ item

Ans: D_2 = 14

Fourth Deciles:

Hence, Rank of D_4,

$= 4\left(\frac{N+1}{10}\right)^{th}$ item

$= 4\left(\frac{19+1}{10}\right)^{th}$ item

$= (4 \text{ X } 2)^{th}$ item

$= 8^{th}$ item

Ans: D_4 = 18

Seventh Deciles:

Hence, Rank of D_7,

$= 7\left(\frac{N+1}{10}\right)^{th}$ item

$= 7\left(\frac{19+1}{10}\right)^{th}$ item

$= (7 \text{ X } 2)^{th}$ item

$= 14^{th}$ item

Ans: D_7 = 24

Q. 2. Find D_1, D_4 and D_8 for the following data:

61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79

Solution: Arranging the values in ascending order, we get,

61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79

The number of values N = 19

First Decile:

Hence, Rank of $D_1 = \left(\frac{N+1}{10}\right)^{th}$ item

$= \left(\frac{19+1}{10}\right)^{th}$ item $= 2^{nd}$ item

Ans: $\mathbf{D_1 = 62}$

Fourth Deciles:

Hence, Rank of $D_4 = 4\left(\frac{N+1}{10}\right)^{th}$ item

$= 4\left(\frac{19+1}{10}\right)^{th}$ item

$= (4 \text{ X } 2)^{th}$ item $= 8^{th}$ item

Ans: $\mathbf{D_4 = 68}$

Eight Deciles:

Hence, Rank of $D_8 = 8\left(\frac{N+1}{10}\right)^{th}$ item

$= (8 \text{ X } 2)^{th}$ item $= 16^{th}$ item

Ans: $\mathbf{D_8 = 76}$

(B) DISCRETE DISTRIBUTION (UNGROUPED FREQUENCY DISTRIBUTION)

Steps:

(1) Arrange Data in ascending order (if not)
(2) Find C.f.
(3) Find rank of required decile.
(4) Determine Decile, based on C.f. value

Example:

Q. 1. Find D_3 and D_4 for following data.

Marks	**1**	**2**	**3**	**4**	**5**	**6**
No. of students	**5**	**6**	**4**	**5**	**10**	**15**

Solution:

Marks	**No. of Students**	**Cumulative Frequency**
1	05	05
2	06	11
3	04	15
4	05	20
5	10	30
6	15	45
	N = 45	

Rank of $D_3 = 3\left(\frac{N+1}{10}\right)^{th}$ item

$D_3 = 3\left(\frac{45+1}{10}\right)^{th}$ item

$D_3 = 3(4.6)^{th}$ item

$D_3 = 13.8^{th}$ item

Hence, D_3 = 3 Marks

Rank of $D_4 = 4\left(\frac{N+1}{10}\right)^{th}$ item

$D_4 = 4\left(\frac{45`+1}{10}\right)^{th}$ item

$D_4 = 4(4.6)^{th}$ item

$D_4 = 18.4^{th}$ item

Hence, D_3 = 3 Marks

(C) CONTINUOUS (GROUPED) DISTRIBUTION:

For calculation of decile in continuous data. We use following formula.

Steps:

(1) Find C.f. **(2)** Find Rank of Decile. **(3)** Apply the formula.

$$D = L_1 + \left(\frac{\frac{N}{10} - C.f}{f}\right) X\ i$$

Where, D = Decile

L1 = lower limit of the decile class.

i = L1 – L2

c.f = Cumulative frequency of preceding class of decile class.

f = frequency of Decile class.

Example:

Q. 1. Calculate the D_2, D_7, and D_9 for the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	4	9	14	25	18	6	2	1	1	0

Solution:

Marks	**No. of Students**	**C.f.**
0-10	04	04
10-20	09	13
20-30	14	27
30-40	25	52
40-50	18	70
50-60	06	76
60-70	02	78
70-80	01	79
80-90	01	80
90-100	00	80
	N = 80	

Decile 2

Rank of $D_2 = \frac{2N}{10} = \frac{2X80}{10} = 16^{th}$

Hence, 20-30 is the class of D2

$L_1 = 20$, i = 10, C.f. = 13, f = 14

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$$D_2 = L_1 + \left(\frac{\frac{2N}{10} - C.f.}{f}\right) x\ i$$

$$= 20 + \left(\frac{\frac{2 \times 80}{10} - 13}{14}\right) \times 10$$

$$= 20 + \left(\frac{16 - 13}{14}\right) \times 10$$

$$= 20 + \left(\frac{3}{14}\right) \times 10$$

$$= 20 + \frac{30}{14}$$

$$= 20 + 2.14$$

Ans: D_2 = 22.14

Rank of $D_7 = \frac{7N}{10} = \frac{7 X 80}{10} = 56$

Hence, 40-50 is the class of D_7

$L_1 = 40$, i = 10, C.f. = 52, f = 18

$$D_7 = L_1 + \left(\frac{\frac{7N}{10} - C.f.}{f}\right) x\ i$$

$$= 40 + \left(\frac{56 - 52}{18}\right) \times 10$$

$$= 40 + \left(\frac{4}{18}\right) \times 10$$

$$= 40 + \frac{40}{18}$$

$$= 40 + 2.22$$

Ans: D_7 = 42.22

Rank of $D_2 = \frac{2N}{10} = \frac{2X80}{10} = 16^{th}$

Hence, 20-30 is the class of D2

$L_1 = 20$, i = 10, C.f. = 13, f = 14

$$D_9 = L_1 + \left(\frac{\frac{9N}{10} - C.f.}{f}\right) x\ i$$

$$= 50 + \left(\frac{72 - 70}{6}\right) \times 10$$

$$= 50 + \left(\frac{2}{6}\right) \times 10$$

$$= 50 + 3.33$$

Ans: D_9= 53.33

Q. 2: Find D_5 for the following distribution.

X	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
F	8	12	20	34	30	25	17	4

Solution:

X	f	**C.**f
0-10	08	08
10-20	12	20
20-30	20	40
30-40	34	74
40-50	30	104
50-60	25	129
60-70	17	146
70-80	04	150
	N = 150	

Rank of $D_5 = \frac{5N}{10} = \frac{5 \times 150}{10} = 75$

Hence, 40-50 is the class of D_5

$L_1 = 40$, C.f. = 74, f = 30

$$D_5 = L_1 + \left(\frac{\frac{5N}{10} - C.f.}{f}\right) x\ i$$

$$= 40 + \left(\frac{75-74}{30}\right) x\ 10$$

$$= 40 + \left(\frac{1}{30}\right) x\ 10$$

$$= 40 + 0.33$$

Ans: D_5 = 40.33

PERCENTILES

Definition: Percentiles divide the series into 100 equal parts. There are 99 percentiles giving ninety none dividing points, the value of which is called percentiles.

There are 99 percentiles in a distribution, the 50^{th} percentiles being the median or second quartile or 5^{th} deciles.

They are denoted by P_1, P_2, P_3 .... P_{99}

SYMBOLICALLY:

First Percentile: P_1 = size of $\left(\frac{N+1}{100}\right)^{th}$ item **Or** $\left(\frac{n}{100}\right)^{th}$ item

Second Percentile: P_2 = size of $2\left(\frac{N+1}{100}\right)^{th}$ item **Or** $\left(\frac{2n}{100}\right)^{th}$ item

Third Percentile: P_3 = size of $3\left(\frac{N+1}{100}\right)^{th}$ item **Or** $\left(\frac{3n}{100}\right)^{th}$ item

Ninety-ninth Percentile: P_{99} = size of $99\left(\frac{N+1}{100}\right)^{th}$ item **Or** $\left(\frac{99n}{100}\right)^{th}$ item

(A) INDIVIDUAL DATA

Steps:

(1) Find Rank of concerned percentile.

(2) Based on result, count the number and determine the percentile.

Example: Calculate 50th percentile for the following data.

10, 20, 30, 40, 50, 60, 70, 80, 90

Solution:

$$\text{Rank of } P_{50} = 50\left(\frac{n+1}{100}\right)^{th} \text{ item}$$

$$P_{50} = 50\left(\frac{9+1}{100}\right)^{th} \text{ item}$$

$$P_{50} = \frac{50 \text{ X } 10^{th}}{} \text{ item}$$

$P_{50} = 5^{th}$ item

5^{th} item is 50

Hence, $P_{50} = 50$

(B) DISCRETE DISTRIBUTION (UNGROUPED DATA):

Steps:

(1) Find C.f.

(2) Find the rank of concerned percentile.

(3) Based on c.f. and rank determine the percentile.

Example: Calculate P_{60} for the following

Marks	10	20	30	40	50
No. of Students	1	6	3	4	5

Solution:

Marks	**No. of Students**	**C.f.**
10	1	01
20	6	07
30	3	10
40	4	14
50	5	19
	N = 19	

$$\text{Rank of } P_{60} = \text{Size of } 60\left(\frac{n+1}{100}\right)^{th} \text{ item}$$

$$P_{60} = 60\left(\frac{19+1}{100}\right)^{th} \text{ item}$$

$$P_{60} = \left(\frac{20}{100}\right)^{th} \text{ item}$$

$P_{60} = 12^{th}$ item

12^{th} item lies in C.f. = 14

Hence, $P_{60} = 40$

(C) CONTINUOUS DISTRIBUTION (GROUPED DATA):

Example: From the following frequency distribution find P_{40} and P_{80}.

Marks	10-20	20-30	30-40	40-50	50-60

No. of Students	5	4	8	4	4

Solution:

Marks	**No. of Students**	**C.f.**
10-20	5	05
20-30	4	09
30-40	8	17
40-50	4	21
50-60	4	25
	N = 25	

Here, N = 25

Rank of $P_{40} = 40\left(\frac{N}{100}\right)^{th}$ value

$= 40\left(\frac{25}{100}\right)^{th}$ value

= 40 x ¼th value

= 10th value

This lies in the class 30-40

$L_1 = 30$, C.f.= 9, $f = 8$, i = 10

$\therefore P_{40} = L_1 + \frac{40\left(\frac{n}{100}\right)}{f}$ - C.f x i

$= 30 + \frac{40\left(\frac{n}{100}\right)}{f} \times 10$

$= 30 + \left(\frac{10-9}{8}\right) \times 10$

$= 30 + \frac{1}{8} \times 10$ $\quad$ = 30+1.25

$P_{40} = 31.25$

Rank of $P_{80} = 80\left(\frac{N}{100}\right)^{th}$ value

$= 80\left(\frac{25}{100}\right)^{th}$ value

$= 80 \times \left(\frac{1}{4}\right)^{th}$ value

= 20th value

This lies in the class 40-50

$L_1 = 40$, C.f.= 17, $f = 4$, i = 10

$\therefore P_{80} = L_1 + \frac{80\left(\frac{n}{100}\right)}{f}$ - C.f x i

$= 40 + \left(\frac{20-17}{4}\right) \times 10$

$= 40 + \frac{3}{10} \times 10$ $\quad = 40 + \frac{30}{4}$

$= 40 + 7.5$

$\mathbf{P_{80} = 47.5}$

SUMMARY:

Central Tendency:-

It may be defined as the tendency of a given set of observation to cluster around a single central or middle value.

And this single value known as Measure of Central Tendency or location or average.

Different Measures of Central tendency:-

1) Arithmetic Mean (AM)
2) Median (Me)
3) Mode (Mo/Z)
4) Geometric Mean (GM)
5) Harmonic Mean (HM)

Criteria for an Ideal Measure of central tendency:-

1) It should be properly and unambiguously defined.
2) It should be easy to comprehend.
3) It should be simple to compute.
4) It should based on all observations.
5) It should have certain desirable mathematical properties.
6) It should be least affected by the presence of extreme observations.

Arithmetic Mean (AM):–

It is defined as the sum of observations to the number of observations.

$$\bar{x} = \frac{\Sigma x_i}{N}$$

Simple (Ungrouped) frequency distribution:-

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma x_i} = \frac{\Sigma f_i x_i}{N}$$

For Grouped Frequency distribution:-

$$\bar{x} = A + \frac{\Sigma f_i d_i}{N} \times c$$

Where, $\bar{x}$ = AM
N = Total No. of observation
A = Assumed mean
$d_i = \frac{x_i - A}{c}$
c = Class Width

Properties of AM:-

1) If all the observations are equal the AM is that number itself.
2) The algebric sum of deviation from AM is zero.
   i.e. $\Sigma(x_i - \bar{x}) = 0$
   and $\Sigma f_i (x_i - \bar{x}) = 0$
3) The two groups with n_1 and n_2 observations and $\bar{x}_1$ and $\bar{x}_2$ AM, then their combined mean
   $$\bar{x} = \frac{n_1\bar{x} + n_2\bar{x}}{n_1 + n_2}$$

Median:-

It is defined as the middle most value when the observations are arranged in ascending or descending order.

For Discrete Variable:-

Median can be found out by inspection.

For Simple (Ungrouped) Frequency distribution:-

Median can be found out by finding the $\left(\frac{N}{2}\right)^{th}$ observation.

For Grouped Frequency distribution:-

Median can be find out by following formula

$$\textbf{Micro economics} = L_1 + \frac{\frac{N}{2} - f_1}{f_m}$$

Where,

l_1 = Lower limit of median class
N = Total Frequency
f_1 = c.f. of pre – median class
f_m = frequency of median class
c = length or width of median class

Partition Values:-

It may be defined as values dividing a given set of observations into number of equal parts.

Quartiles:-

These are the values which divides the given set of observation into 4 equal parts.

So, there are 3 Quartiles Q_1, Q_2 and Q_3 .

$$Q_p = (n + 1) \times \frac{p}{4}$$

Deciles:-

These are the values which divides the given set of observations into 10 equal parts.

So, there are 9 deciles.

i.e. $D_1, D_2, D_3 \ldots\ldots\ldots D_9$

$$Dp = (n + 1) \times \frac{P}{10}$$

Percentiles or Centiles:-

These are the values which divides the given set of observations into 100 equal parts.

So, there are 99 percentiles.

i.e. P_1 , P_2, P_3 ....P_{99}.

$$PP = (n+ 1) \times \frac{P}{100}$$

Where,

n = Total observations

Mode :-

It is defined as the value that occurs the maximum number of times.

Depending upon the observation values of modes may one or more or none.

For unclassified Data:-

Mode can be find out by inspection.

For Frequency (Ungrouped) distribution:-

Mode is the observation having maximum frequency.

For Frequency (grouped) Distribution:-

Mode can be calculated as,

$$\text{Mode} = l_1 + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times c$$

Where,

l_1 = LCB of modal class

f_0 = frequency of modal class

f_1 = frequency of pre–modal class

f_2 = frequency of post modal class

c = Class length.

Mean – Mode = 3(Mean – Median)

Geometric Mean:-

It is defined as the n^{th} root of the product of the observation.

$G = (x_1, x_1, x_3 \ldots\ldots\ldots x_n)^{1/n}$ **For unclassified data**

$= \left(x_1^{f1}, x_2^{f2}, x_3^{f3}, \ldots\ldots. x_n^{fn}\right)^{1/n}$ .... For frequency distribution

Properties:-

1) Logarithm G for a set of observations is the AM of the logarithm of the observations.
2) If all the observations are equal (say K) then their GM is also K.
3) GM of the product of 2 variable is the product of their GM's if z = xy then (GM) of z = (GM) of x X (GM) of y.
4) GM of the ratio of 2 variables is the ratio of the GM's of the 2 variable.
   i.e. If z = x/y then,
   $$(GM) \text{ of } z = \frac{(GM) \text{ of } x}{(GM) \text{ of } y}$$

Harmonic Mean (HM):-

It is defined as the reciprocal of the AM of the reciprocal of the observations.

$$H = \frac{n}{\Sigma[\frac{1}{x_i}]}$$

Frequency distribution:-

$$H = \frac{N}{\Sigma[\frac{f_i}{x_i}]}$$

Properties:-

1) If all the observations are equal (say k) then the HM is also k.
2) If there are 2 groups with n_1 and n_2 observations and H_1 and H_2 as respective HM's then the combined HM is given by
   $$H = \frac{n_1 + n_2}{\frac{n_1}{H_2} + \frac{n_2}{H_2}}$$
3) Relation between AM, GM and HM.
   For any set of +ve observations
   AM ≥ GM ≥ HM

Properties of GM and HM:-

1) Both possess some mathematical properties.
2) Rigidly defined
3) Based on all observations
4) Difficult to comprehend
5) Difficult to Compute

Select the correct alternative out of the given ones:

1) Measures of central tendency are known as
   (A) Difference
   (B) Averages
   (C) Both
   (D) None of these
2) Measures of central tendency for a given set of observations measures
   (A) The scatter ness of the observation
   (B) The central location of the observations
   (C) Both (A) and (B)
   (D) None of these
3) The average has relevance for
   (A) Homogeneous population
   (B) Heterogeneous population
   (C) Both (A) and (B)
   (D) None of these
4) A measure of central tendency tries to estimate the
   (A) Central value
   (B) Lowe value
   (C) Upper value
   (D) None of these
5) The number of measure of central tendency is
   (A) Two
   (B) Three
   (C) Four
   (D) five
6) A. M. is never less the G.M.
   (A) True
   (B) False
   (C) Both (A) and (B)
   (D) None of these
7) While computing the A.M. from a grouped frequency distribution, we assume that
   (A) The classes are of equal length
   (B) The classes have equal frequency
   (C) All the values of a class are equal to the mid-value of that class
   (D) None of these
8) If the class interval is open-end, then it is difficult to find
   (A) Frequency
   (B) A.M.
   (C) Both (A) and (B)
   (D) None of these
9) The words 'mean' or 'average' only refer to
   (A) A.M
   (B) G.M
   (C) H.M
   (D) None of these
10) Which of the following statements is wrong?
   (A) Mean in rigidly defined
   (B) Mean is not affected due to sampling fluctuations.
   (C) Mean has some mathematical properties.
   (D) All these
11) When all values occur with equal frequency, there is No.
   (A) Mode
   (B) Mean
   (C) Median
   (D) None of these
12) Weighted A.M is related to
   (A) G.M
   (B) Frequency
   (C) H.M
   (D) None of these
13) The most commonly used measure of central tendency is
   (A) A.M
   (B) Median
   (C) Mode
   (D) Both G.M and H.M
14) Weighted averages are considered when
   (A) The data are not classified.
   (B) The data are put in the form of grouped frequency distribution
   (C) All the observations are not of equal importance.
   (D) Both (A) and (C)
15) The average discovers
   (A) Uniformity in variability.
   (B) Variability in uniformity of distribution
   (C) Both (A) and (B)
   (D) None of these
16) Each different values is considered only once for
   (A) Simple average
   (B) Weight average
   (C) Both (A) and (B)
   (D) None of these
17) Which of the following statements is true?
   (A) Usually means is the best measure of central tendency.
   (B) Usually median is the best measure of central tendency.
   (C) Usually mode is the best measure of central tendency.
   (D) Normally, G.M is the best measure of central tendency.
18) When a frequency distribution is given, the frequencies of values are themselves treated as weights.
   (A) True
   (B) False
   (C) Both (A) and (B)

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(D) None of these

19) The sum of the squares of deviations of a set of observations has the smallest value, when the deviations are taken from their
(A) A.M
(B) H.M
(C) G.M
(D) None of these

20) For a given set of observations A.M is greater than G.M
(A) True
(B) False
(C) Both (A) and (B)
(D) None of these

21) Frequencies are generally used as
(A) Range
(B) Weights
(C) Mean
(D) None of these

22) The values of all items are taken into consideration in the calculation of
(A) Median
(B) Mean
(C) Mode
(D) None of these

23) The word 'average' used in 'simple average' and 'weighted average' generally refers to
(A) Median
(B) Mode
(C) A.M or G.M or H.M
(D) None of these

24) The total of a set of observations is equal to the product of their numbers and the
(A) A.M
(B) G.M
(C) A.M
(D) None of these

25) Simple average is sometimes called
(A) Weighted average
(B) Unweighted average
(C) Relative average
(D) None of these

26) The algebraic sum of deviations of observations from their A.M is
(A) 2
(B) -1
(C) 1
(D) 0.

27) G.M is less than H.M
(A) True
(B) False
(C) Both
(D) None of these

28) Which of the following measure of the central tendency is difficult to compute?
(A) Mean
(B) Median
(C) Mode
(D) G.M

29) You are given the population of India for the courses of 1981 and 1991. you are to find the population of India at the middle of the period by averaging these population figures, assuming a consistent rate of increase of population. What is the suitable of average in this case
(A) A.M
(B) G.M
(C) H.M
(D) None of these

30) Which measure(s) of central tendency is(are) considered for finding the average rates?
(A) A.m
(B) G.m
(C) H.m
(D) Both (B) and (C)

31) Calculation of G.M is more difficult than
(A) A.M
(B) H.m
(C) Median
(D) None of these

32) When a firm registers both profits and losses, which of the following measure of central tendency cannot be considered?
(A) A.m
(B) G.m
(C) Median
(D) Mode.

33) G.M is defined only with
(A) All observation have the same sign and none is zero
(B) All observations have the different sign and none is zero
(C) All observations have the same sign and one is zero
(D) All observation have the different sign and one is zero

34) More laborious numerical calculations involves in G.M than A.M
(A) True
(B) False
(C) Both
(D) None of these

35) G.M is useful in construction of index number
(A) True
(B) False

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(C) Both
(D) None of these

36) In .... the quantities are in ratios.
(A) A.M
(B) G.M
(C) H.M
(D) None of these

37) .... is not mush affected by fluctuations of sampling
(A) A.M
(B) G.M
(C) H.M
(D) None of these

38) .... is the most stable of all the measures of central tendency
(A) G.M
(B) H.M
(C) S.M
(D) None of these

39) .... of a set of observations is defined to be their sum, divided by the number of observations.
(A) H.M

(B) G.M
(C) A.M
(D) None of these

40) .... is used when variability has also to be calculated.
(A) A.M
(B) G.M
(C) H.M
(D) None of these

41) Logarithm of G.M is the .... of the different values.
(A) Weighted mean
(B) Simple mean
(C) Both
(D) None of these

42) Mean is of .... types.
(A) 3
(B) 4
(C) 8
(D) 5.

43) .... average is obtained on dividing the total of a set of observations by their number
(A) Simple
(B) Weighted
(C) Both
(D) None of these

44) .... is useful in averaging ratios, rates and percentages.
(A) A.M
(B) G.M
(C) H.M
(D) None of these

45) G.M of a set on n observations is the .... root not their product.
(A) $(n/2)^{th}$
(B) $(n+1)^{th}$
(C) n^{th}
(D) $(n-1)^{th}$.

46) .... and .... cannot be calculated if any observation is zero.
(A) G.M & A.M
(B) H.M & A.M
(C) H.M & G.M
(D) None of these

47) .... has a limited use
(A) A.M
(B) G.M
(C) H.M
(D) None of these

48) .... is the reciprocal of the A.M of reciprocal of observations.
(A) H.M
(B) G.M
(C) Both
(D) None of these

49) .... is used when the sum of deviations from the average should be least.
(A) Mean
(B) Mode
(C) Median
(D) None of these

50) Half of the numb3er in an ordered set have values less than the .... values greater than the ....
(A) Mean, median
(B) Median, median
(C) Mode, mean
(D) None of these

51) In the case of continuous frequency distribution the size of the .... item indicates class interval in which the median lies.
(A) (n-1)/2th
(B) (n+1)/2th
(C) (n/2)th
(D) None of these

52) .... is a good substitute to a weighted average.
(A) A.M
(B) G.M
(C) H.M
(D) None of these

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53) .... is used when rate of growth or decline is required.
(A) Mode
(B) A.M
(C) G.M
(D) None of these

54) The deviations from median are .... it negative signs are ignored as compared to other measures of central tendency.
(A) Minimum
(B) Maximum
(C) Same
(D) None of these

55) The middle most value of a set of observations is
(A) Median
(B) Mode
(C) Both
(D) None of these

56) Median is unaffected by extreme values.
(A) True
(B) False
(C) Both
(D) None of these

57) In case of an even number of observations which of the following is median?
(A) Any of the two middle-most value
(B) The simple average of these two middle values
(C) The weighted average of those two middle values
(D) Any of these

58) The value of the middle –most item when they are arranged in order of magnitude is called
(A) Standard deviation
(B) Mean
(C) Mode
(D) Median

59) When the distribution is symmetrical, mean, median and mode
(A) Coincide
(B) Do not coincide
(C) Both
(D) None of these

60) For open-end classification, Which of the following is the best measure of central tendency
(A) Median
(B) Mean
(C) Mode
(D) None of these

61) The values of extreme items do not influence the average in case of
(A) Median
(B) Mean
(C) Mode
(D) None of these

62) The presence of extreme observations does not affect
(A) A.M
(B) Median
(C) Mode
(D) Any of these

63) In a distribution with a single peak and moderate skewness to the right it is closer to the concentration of the distribution in case of
(A) Mean
(B) Median
(C) Both
(D) None of these

64) For an even number of values the median is the
(A) Average of two middle values
(B) Middle value
(C) Both (A) and (B)
(D) None of these

65) For a moderately skewed distribution, which of the following relationship holds?
(A) Mean-Mode=3(Mean-Median)
(B) Median-Mode=3(Mean-Median)
(C) Mean-Median=3(Mean-Mode)
(D) Mean-Median=3(Median-Mode)

66) In .... the distribution has wide range of variations.
(A) Median
(B) Mode
(C) Mean
(D) None of these

67) .... is used when distribution pattern has to be studied at varying levels.
(A) A.M
(B) Median
(C) G.M
(D) None of these

68) .... is used when representative value is required and distribution is asymmetric
(A) Mode
(B) Mean
(C) Median
(D) None of these

69) .... always lies in between the arithmetic means and mode.
(A) G.M
(B) H.M
(C) Median

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(D) None of these

70) 50% of actual values will be below and 50% of will be above ....
(A) Mode
(B) Median
(C) Mean
(D) None of these

71) In .... the distribution has open-end classes.
(A) Median
(B) Mean
(C) Standard
(D) None of these

72) .... can be calculated from a frequency distribution with open end intervals
(A) Median
(B) Mean
(C) Mode
(D) None of these

73) .... is equal to the value corresponding to cumulative frequency
(A) Mode
(B) Mean
(C) Median
(D) None of these

74) .... is the value of the variable corresponding to cumulative frequency N/2
(A) Mode
(B) Mean
(C) Median
(D) None of these

75) .... divides the total number of observations into two equal parts.
(A) Mode
(B) Mean
(C) Median
(D) None of these

76) .... is called a positional measure.
(A) Mode
(B) Mean
(C) Median
(D) None of these

77) The number of observation smaller than .... is the same as the number larger than it.
(A) Median
(B) Mean
(C) Mode
(D) None of these

78) The value of a variate that occur most often is called
(A) Median
(B) Mean
(C) Mode
(D) None of these

79) The class having maximum frequency is called
(A) Modal class
(B) Median class
(C) Mean class
(D) None of these

80) The value with occurs with the maximum frequency is called
(A) Median
(B) Mode
(C) Mean
(D) None of these

81) Which of the following measure(s) satisfied (satisfy) a liner relationship between two variables?
(A) Mean
(B) Median
(C) Mode
(D) All of these

82) Which one of the following is not uniquely defined?
(A) Mean
(B) Median
(C) Mode
(D) All these

83) Which of the following measure(s) possess mathematical properties?
(A) A.M
(B) G.M
(C) H.M
(D) All of these

84) For determination of mode, the class intervals should be
(A) Overlapping
(B) Maximum
(C) Minimum
(D) None of these

85) Relation between mean, median and mode is
(A) Mean-mode=2(mean...median)
(B) Mean-median=2(mean...mode)
(C) Mean-median=2(mean...mode)
(D) Mean-mode=2(mean...median)

86) For the calculation of................ the data must be arranged in the form of a frequency distribution
(A) Median
(B) Mode
(C) Mean
(D) None

87) .... is used when most frequency occurring value is required (discrete variables).

(A) Mode
(B) Mean
(C) Median
(D) None of these

88) For calculation of .... we have to construct a grouped frequency distribution
(A) Median
(B) Mode
(C) Mean
(D) None of these

89) .... is used when sampling variability should be least.
(A) Mode
(B) Median
(C) Mean
(D) None of these

90) .... is the value of the variable corresponding to the highest frequency
(A) Mode
(B) Mean
(C) Median
(D) None of these

91) For ordering shoes of various sizes for resale, a .... size will be more appropriate.
(A) Median
(B) Modal
(C) Mean
(D) None of these

92) .... cannot be related a algebraically.
(A) Mode
(B) Mean
(C) Median
(D) None of these

93) Extreme values have .... effect on mode.
(A) High
(B) Low
(C) No
(D) None of these

94) Measures which are used to divide or partition, the observations into a fixed number of parts are collectively known as
(A) Partition values
(B) Quartiles
(C) Both
(D) None of these

95) There are .... quartiles
(A) 1
(B) 3
(C) 2
(D) 4.

96) Quartiles are the values which divide a given set of observations into
(A) Two equal parts
(B) Four equal parts
(C) Five equal parts
(D) None of these

97) The second quartile is known as
(A) Median lower quartile
(B) Upper quartile
(C) None of these

98) Corresponding to second quartile, the cumulative frequency is
(A) N/4
(B) 2 N/4
(C) 3 N/4
(D) None of these

99) The values which divide the total number of observations into 10 equal parts are
(A) Quartiles
(B) Percentiles
(C) Deciles

100) .... divide the total number observation into 4 equal parts.
(A) Median
(B) Deciles
(C) Quartiles
(D) Percentiles

101) Lower quartile is
(A) First quartile
(B) Second quartile
(C) Upper quartile
(D) None of these

102) .... quartile is known as Upper quartile
(A) First
(B) Second
(C) Third
(D) None of these

103) The lower and upper quartiles are used to define.
(A) Standard deviation
(B) Quartile deviation
(C) Both (A) and (B)
(D) None of these

104) Between second and upper quartile, the frequency is equal to
(A) 3 N/4
(B) N/4
(C) N/2
(D) None of these

105) Corresponding to upper quartile, the cumulative frequency is
(A) 3 N/4
(B) 2 N/4
(C) N/2
(D) None of these

106) For grouped frequency distribution .... is equal to the value corresponding to cumulative frequency N/4
(A) Median
(B) 1^{st} quartile
(C) 3^{rd} quartile
(D) None of these

107) .... are used for measuring central tendency, dispersion and skewness.
(A) Median
(B) Deciles
(C) Percentiles
(D) quartiles

108) Quartiles can be determined graphically using
(A) Histogram
(B) Frequency Polygon
(C) Ogive
(D) Pie chart.

109) Between first and second quartile, the frequency is equal to
(A) 3 N/4
(B) N/2
(C) N/4
(D) None of these

110) For grouped frequency distribution .... is equal to the value corresponding to cumulative frequency 3N/4
(A) Median
(B) 1^{st} quartile
(C) 3^{rd} quartile
(D) None of these

111) Ninth Decline lies in the class interval of the
(A) (n/9)th
(B) (9n/10)th
(C) (9n/20)th
(D) None of these

112) There are .... deciles.
(A) 7
(B) 8
(C) 9
(D) 10

113) Fifth decline is equal to
(A) Mode
(B) Median
(C) Mean
(D) None of these

114) For grouped frequency distribution .... is equal to the value corresponding to cumulative frequency kN/10
(A) Median
(B) *k*th percentile
(C) *k*th decile
(D) None of these

115) .... is equal to the value corresponding to cumulative frequency k(N+1)/10 from simple frequency distribution
(A) Median
(B) kth decile
(C) kth percentile
(D) None of these

116) The values which divide the total number of observations into 100 equal parts is
(A) Percentiles
(B) Quartiles
(C) Deciles
(D) None of these

117) 50^{th} percentile is known as
(A) 50^{th} decile
(B) 50^{th} quartile
(C) Mode
(D) Median.

118) 25^{th} percentile is equal to
(A) 1^{st} quartile
(B) 25^{th} quartile
(C) 24^{th} quartile
(D) None of these

119) Calculation of quartiles, deciles percentiles may be obtained graphically from
(A) Frequency Polygon
(B) Histogram
(C) Ogive
(D) None of these

120) .... is equal to the value corresponding to cumulative frequency k(N+1)/100 from simple frequency distribution
(A) kth decile
(B) kth percentile
(C) Both
(D) None of these

121) For the values of a variable 5, 2, 8, 3, 7, 4, the median is:
(A) 4
(B) 4, 5
(C) 5
(D) None of these

122) Variable: 2 3 4 5 6 7
No. of men: 5 6 8 13 7 4

mode is:
(A) 6
(B) 4

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(C) 5 (D) None of these

ANSWERS:

1.	B	**2.**	B	**3.**	B	**4.**	A	**5.**	B	**6.**	A	**7.**	C	**8.**	B
9.	A	**10.**	B	**11.**	B	**12.**	B	**13.**	A	**14.**	C	**15.**	A	**16.**	A
17.	A	**18.**	C	**19.**	A	**20.**	A	**21.**	B	**22.**	B	**23.**	C	**24.**	C
25.	B	**26.**	D	**27.**	B	**28**	D	**29**	B	**30**	D	**31**	A	**32.**	B
33.	A	**34.**	A	**35.**	A	**36.**	A	**37.**	B	**38.**	C	**39.**	C	**40.**	A
41.	A	**42.**	A	**43.**	A	**44.**	B	**45.**	C	**46.**	C	**47.**	C	**48.**	A
49.	A	**50.**	B	**51.**	C	**52.**	C	**53.**	C	**54.**	A	**55.**	A	**56.**	A
57.	B	**58.**	D	**59.**	A	**60.**	C	**61.**	A	**62.**	B	**63.**	B	**64.**	A
65.	A	**66.**	A	**67.**	B	**68.**	C	**69.**	C	**70.**	B	**71.**	A	**72.**	A
73.	C	**74.**	C	**75.**	C	**76.**	C	**77.**	A	**78.**	C	**79.**	A	**80.**	B
81.	D	**82.**	C	**83.**	D	**84.**	A	**85.**	D	**86.**	B	**87.**	A	**88.**	B
89.	A	**90.**	A	**91.**	B	**92.**	A	**93.**	C	**94.**	C	**95.**	B	**96.**	B
97.	A	**98.**	B	**99.**	C	**100.**	C	**101.**	A	**102.**	C	**10.3**	B	**104.**	B
105.	A	**106.**	B	**107.**	D	**108.**	C	**109.**	C	**110.**	C	**111.**	B	**112.**	C
113.	B	**114.**	C	**115.**	B	**116.**	A	**117.**	D	**118.**	A	**119.**	C	**120.**	B
121.	B	**122.**	C												

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Write done the correct answer out of the given ones:

1) The algebraic sum of deviations of 8, 1, 6 from the A.M. viz., 5 is
   (A) -1
   (B) 0
   (C) 1
   (D) None of these
2) G.M. of 8, 4, 2 is:
   (A) 4
   (B) 2
   (C) 8
   (D) None of these
3) Mean of 0.3, 5, 6, 7, 9, 12, 0.2 is:
   (A) 4.9
   (B) 5.7
   (C) 5.6
   (D) None of these
4) Mode of 0, 3, 5, 6, 7, 9, 12, 0.2 is:
   (A) 6
   (B) 0
   (C) 3
   (D) 5
5) Mode of 40, 50, 30, 20, 25, 35 30, 30, 20, 30 is:
   (A) 25
   (B) 30
   (C) 35
   (D) None of these
6) Median of 2, 5, 8, 4, 9, 6, 7 is:
   (A) 9
   (B) 8
   (C) 8
   (D) 6
7) Mode of the observations 2, 5, 8, 4, 3, 4, 4, 5, 2, 4, 4 is:
   (A) 3
   (B) 2
   (C) 5
   (D) 4
8) For the observations 5, 3, 6, 3, 5, 10, 7, 2 there are .... modes.
   (A) 2
   (B) 3
   (C) 4
   (D) 5
9) Mode of 15, 12, 5, 13, 12, 15, 8, 8, 9, 9, 10, 15 is:
   (A) 15
   (B) 12
   (C) 8
   (D) 9
10) Mode of 40, 50, 30, 20, 25, 35, 30, 30, 20, 30 is:
   (A) 25
   (B) 30
   (C) 35
   (D) None of these
11) What is the median for the following observations? 5, 8, 6, 9, 11, 4.
   (A) 6
   (B) 7
   (C) 8
   (D) None of these
12) If there are 3 observation 15, 20, 25 then the sum of deviation A.M. is:
   (A) 0
   (B) 5
   (C) -5
   (D) None of these
13) What if the modal value for the numbers 5, 8, 6, 4, 10, 15, 18, 10?
   (A) 10
   (B) 6
   (C) 18
   (D) None of these
14) The harmonic mean for the numbers 3, 4, 5 is:
   (A) 2.00
   (B) 3.33
   (C) 3.83
   (D) 4.83
15) The mode of the numbers 7, 7, 7, 9, 10, 11, 11, 11, 12 is:
   (A) 11
   (B) 12
   (C) 7
   (D) 7 & 11
16) What is the G.M. for eh numbers 8, 24 and 40?
   (A) 24
   (B) 12
   (C) $8\sqrt[3]{15}$
   (D) 10
17) If y = 3x – 100 and $\bar{\chi}$ = 50, then the value of $\bar{\gamma}$ is:
   (A) 60
   (B) 30
   (C) 100
   (D) 50
18) If y = 5x – 20 and $\bar{\chi}$ = 30, then the value of $\bar{\gamma}$ is:
   (A) 130
   (B) 140

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(C) 30
(D) None of these

19) The median of the numbers 11, 10, 12, 13, 9 is:
(A) 11
(B) 12
(C) 10.5
(D) 12.5

20) The A.M. of 1, 3, 5, 6, x, 10 is 6. the value of x is:
(A) 10
(B) 11
(C) 12
(D) None of these

21) Two variables x and y are given by y = 2x – 3. If the median of x is 20, what is the median of y?
(A) 20
(B) 40
(C) 37
(D) 35

22) If a variable assumes the values 1, 2, 3, 4, 5 with frequencies as 1, 2, 3, 4, 5, then what is the A.M.?
(A) $\frac{11}{3}$
(B) 5
(C) 4
(D) 4.50

23) If the A.M. and G.M. for two numbers are 6.50 and 6 respectively, then the two numbers are:
(A) 6 and 7
(B) 9 and 4
(C) 10 and 3
(D) 8 and 5.

24) If there are two groups containing 30 and 20 observations and having 50 and 60 as arithmetic means, then the combined arithmetic mean is:
(A) 55
(B) 54
(C) 56
(D) 52

25) What is he H.M. of 1, ½, 1/3, ... 1/n?
(A) n
(B) 2n
(C) $\frac{2}{(n+1)}$
(D) $\frac{n(n+1)}{2}$

26) If x and y are related by x-y-10=0
(A) 20
(B) 13
(C) 3
(D) 23

27) And mode of x is known to be 23, then the mode of y is:
(A) 13
(B) 10.70
(C) 11
(D) 11.50

28) If he relationship between two variables u and v are given by 2u+v+7=0 and if the A.M. of u is 10, then the A.M. of v is:
(A) 17
(B) -17
(C) -27
(D) 27

29) An aeroplanes files from A to B at the rate of 500 km/hour and comes back from B to A at the rate of 700 km/hour. The average speed of the aeroplanes is:
(A) 600 km, per hour
(B) 583.33 km. per hour
(C) $100\sqrt{35}$ km per hour
(D) 620 km per hour

30) If the A.M. and G.M. for 10 observations are both 15, then the value of H.M. is
(A) Less than 15
(B) More than 15
(C) 15
(D) Can not be determined.

31) The mean salary for a group of 40 female workers is Rs. 5200 per month and that for a group of 60 male workers is Rs. 6800 per month. What is the combined salary?
(A) Rs. 6160
(B) Rs. 6260
(C) Rs. 6560
(D) Rs. 6760

32) The average salary of a group of unskilled workers is Rs. 10000 and that of a group of skilled workers is Rs. 15,000. if the combined salary is Rs. 12,000 then what is the percentage of skilled workers?
(A) 40%
(B) 50%
(C) 60%
(D) None of these

Write down the correct answers. Each question carries 5 marks.

33) What is the value of mean and median for the following data:

Marks:	5-14	15-24	25-34	35-44	45-54	55-64
No. of Student:	10	18	32	26	14	10

(A) 30 and 28
(B) 29 and 30
(C) 33.68 and 32.94
(D) 34.21 and 33.18

34) The mean and mode for the following frequency distribution

Class Interval:	350-369	370-389	390-409	410-429	430-449	450-469
Frequency:	15	27	31	19	13	6

Are:
(A) 400 and 390
(B) 400.58 and 390
(C) 400.58 and 394.50
(D) 400 and 394.

35) The third quartile and 65th percentile for the following data:

Profits in '000 Rs:	less than 10	10-19	20-29	30-39	40-49	50-59
No. of firms:	5	18	38	20	9	2

Are:
(A) Rs. 33500 and Rs. 29184
(B) Rs. 33000 and Rs. 28680
(C) Rs. 33600 and Rs. 29000
(D) Rs. 33250 and Rs. 29250

36) Following is an incomplete distribution having modal mark as 44

Mars	0-20	20-40	40-60	60-80	80-100
No. of Students	5	18	?	12	5

What would be the mean marks?
(A) 45
(B) 46
(C) 47
(D) 48

37) The data relating to the daily wage of 20 workers are shown below:

Rs. 50, Rs. 55, Rs. 60, Rs. 58, Rs. 59, Rs. 72, Rs. 65, Rs. 68, Rs. 53 Rs. 50, Rs. 67, Rs. 58, Rs. 63, Rs. 69, Rs. 74, Rs. 63, Rs. 61, Rs. 57, Rs. 62, and Rs. 64.

The employer pays bonus amounting to Rs. 100, Rs. 200, Rs. 300, Rs. 400 and Rs. 500 to the wage earners in the wage groups Rs. 50 and not more than Rs. 55 and not more than Rs. 60 and so on and lastly Rs. 70 and not more than Rs. 75, during the festive month of October.

What is the average bonus paid per wage earner?
(A) Rs. 200
(B) Rs. 250
(C) Rs. 285
(D) Rs. 270

38) What is the value of the first quartile for observations 15,18, 10, 20, 23, 28, 12, 16?
(A) 17
(B) 16
(C) 12.75
(D) 12

39) If the A.M and H.M for two numbers are 5 and 3.2 respectively then the G.M. will be:
(A) 16.00
(B) 4.10
(C) 4.05
(D) 4.00

40) If there are two groups with 75 and 65 s harmonic means and containing 15 and 13 observation, then the combined H.M is given by:
(A) 65
(B) 70.36
(C) 70
(D) 71

41) If G.M of x is 10 and G.M of y is 15 then the G.M of xy is
(A) 150
(B) Log 10 x log 15
(C) Log 150
(D) None of these

42) The G.M of 4, 6 and 8 is:
(A) 4.77
(B) 5.77
(C) 6.21
(D) 6.77

ANSWERS:

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1.	B	**2.**	A	**3.**	B	**4.**	B	**5.**	B	**6.**	D	**7.**	D	**8.**	A
9.	A	**10.**	B	**11.**	B	**12.**	A	**13.**	A	**14.**	C	**15.**	D	**16.**	C
17.	D	**18.**	A	**19.**	A	**20.**	B	**21.**	C	**22.**	A	**23.**	B	**24.**	B
25.	C	**26.**	B	**27.**	B	**28**	C	**29**	B	**30**	C	**31**	A	**32.**	A
33.	C	**34.**	C	**35.**	A	**36.**	D	**37.**	D	**38.**	C	**39.**	D	**40.**	C
41.	A	**42.**	B												

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